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Developed in these materials is a concept of measurement. The unit begins with a fictitious system of measurement from which basic ideas about measurement are to be retained and later applied to other systems of measurement. It is hypothesized that it is easier for the student to abstract principles from a less tangible and unfamiliar system than from an endless sequence of systems. The initial contact with measurement is also made more exciting and entertaining than can be accomplished by a discourse on yards, feet, and inches. Several parts of this unit are conventional in approach. This work was prepared under ESEA Title III contract. (RP)

CILAMP

Central Iowa Low Achiever Mathematics Project

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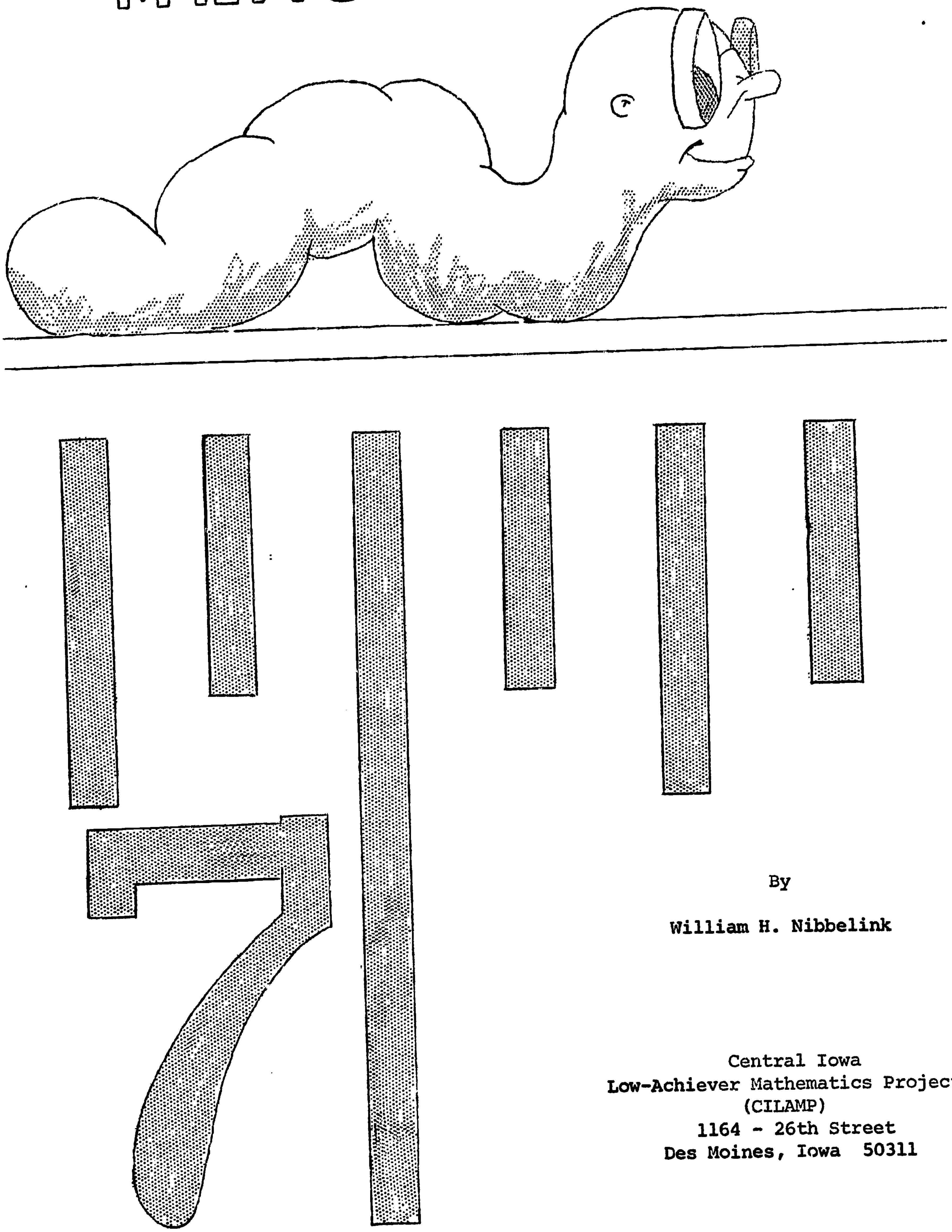
MEASUREMENT



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COMMENTS

MEASUREMENT

The approach used in the unit on measurement is not as unorthodox as it may appear to be at first glance. If it is different from what students are accustomed to seeing, it is intentionally different on only two points.

1) Instead of beginning by studying a familiar system of measurement and then studying other familiar systems of measurement, this unit begins with a fictitious system of measurement from which basic ideas about measurement will hopefully be retained and later applied to other systems of measurement. The reasoning behind this approach is that it will be easier for the student to abstract these principles from a less tangible and much more unfamiliar system than from an endless sequence of systems which the student believes to be pretty much unrelated to each other.

2) The second point of difference is an attempt to make the initial contact with measurement slightly more exciting and entertaining than can be accomplished by a discourse on yards, feet, and inches.

A large part of the unit is very traditional in approach, in some cases because traditional approaches seemed satisfactory and in other cases because of a weariness which comes with attempting to be creative and innovative. If you believe that the first parts of the book are somewhat exciting, you will find that the unit slowly progresses from that point to a more traditional boredom. (To insure a degree of success in presenting this material, the teacher should try not to make any growing boredom too apparent to the students who have most likely been bored by measurement for years. There has never yet been a unit which could succeed without the teacher's cooperation and this unit is certainly no exception. On the other hand, there have been many teachers who have succeeded in spite of a book or unit.)

If the unit is to be used with low achievers, the reading should not be entrusted to the students. The fact that the material is being handed out in a mathematics class is reason enough to leave it alone for most of these students.

It is also the writer's opinion that "grades" on the exercises should be used as a very last resort and only on students who have been irretrievably conditioned to the principle that the only valid demand for effort is the thread of a "bad grade." Most students who are asked to look at this unit will have had experience with measurement at an earlier time and will appreciate any change in approach which will allow memory to rest. In other words, you are being encouraged to have as much fun with this unit as is appropriate to learning, classroom wall thickness, administration attitudes toward experiment, etc., etc.

If the students have a degree of confidence with measurement, it may be advisable to look at pages twenty-five through twenty-eight before beginning the unit.

There are more exercises than many classes will need. The teacher should feel free to select in the best interest of the students.

--William H. Nibbelink

ANSWERS TO EXERCISES

It would be pretentious and unexciting to assure you that all the answers appearing here are correct. In fact, it is sometimes a healthy thing to have a few wrong answers since there are few classroom joys which exceed proving the answer book wrong. None of the answers are intentionally wrong, but the writer's experience places a high probability on at least one or more of the answers being wrong.

Since very little thought was given to making problems easy to find and refer to, the answers will be given with a page number and further hints attempting to describe which part of the page is being referred to.

3-1 308 squints

3-2 37 squints

3-3 1 squint

3-4 0 squints

3-5 101 or 102 squints

5-6 6, 12

5-7 48, 54, 60

5-8 part c

5-9 8 below 48, 9 below 54, 10 below 60

5-10 part b

6-11-a 1 thrump

6-11-b 18 squints

6-11-c 3 thrumps

6-11-d 12 squints

6-11-e 0 thrumps

6-11-f 3 thrumps

6-12 b,c,d

6-13 f

7-13 part b

7-14 part c

7-15-a 2 thrumps

7-15-b 3 thrumps

7-15-c 12 squints

7-15-d 26 thrumps

7-15-e 180 squints

7-15-f 2 thrumps

7-15-g 2 thrumps

7-16 no

9-1 no

10-2 no

10-3 yes

10-4 yes

10-5 no

10-6-1 1 thrumprin

10-6-2 5 thrumprins, 2 squins

10-6-3 OK

10-6-4 OK

10-6-5 3 thrumprins, 1 squin

10-7 5

11-8 part c

12-9-a 2 thrumps, 2 squints

12-9-b 8 thrumps, 0 squints

12-9-c 0 thrumps, 5 squints

12-9-d 111 thrumps, 0 squints

12-9-e 8 thrumps, 2 squints

12-9-f 1 thrump, 0 squints

12-9-g 0 thrumps, 4 squints

12-9-h 111 thrumps, 3 squints

12-11 part c

15-top-1 1 thrump, 1 squint
2 1 thrump, 2 squints
3 10 thrumps
4 10 thrumps, 1 squint
5 10 thrumps, 2 squints
6 1 thrump
7 1 squint

15-top-8 100 thrumps
9 116 thrumps, 4 squints

15-bottom 0 below 0
1 below 5
2 below 10
3 below 15
4 below 20
10 below 50
11 below 55
12 below 60
13 below 65

16-top-1 5 squints
2 25 squints
3 15 squints
4 30 quints

16-middle same as flow-chart on page 14 with "5" replacing "6"

16-bottom-1 1 thrump, 2 squints
2 1 thrump, 1 squint
3 4 thrumps, 3 squints
4 10 thrumps, 0 squints
5 50 thrumps, 1 squint

17-top (1 thrump = 12 squints)

thrump numbers g below 12, 24, 36, 48

17-middle same as flow-chart on page 14 with "12" replacing "6"

17-bottom-1 12 squints
2 144 squints
3 24 squints
4 120 squints

18-middle-1 1 thrump, 1 squint
2 2 thrumps, 0 squints
3 3 thrumps, 8 squints
4 3 thrumps, 11 squints
5 4 thrumps, 0 squints
6 4 thrumps, 1 squint
7 4 thrumps, 2 squints

19-top same as flow-chart on page 14 with "12" replacing "6"

19-bottom-1 1 thrump, 2 squints
2 0 thrumps, 5 squints
3 10 thrumps, 0 squints
4 10 thrumps, 1 squint
5 10 thrumps, 2 squints

20-bottom "thumps," "squints," "6"

22-1-a 48 inches

22-1-b 15 yards

22-1-c 3520 yards

22-1-d 4 feet, 8 inches

22-1-e 3 yards, 2 feet

22-1-f 1 mile, 240 yards

22-2-a 50 chains

22-2-b 7 furlongs, 2 chains

22-2-c 10 furlongs

22-3-a 400 links

22-3-b 357 chains

22-4 7920 inches (or 660 feet)

23-5-a 30 feet

23-5-b 3 feet

23-5-c 300 fathoms

23-5-d 1800 feet

23-5-e 1800 feet

23-5-f 4 fathoms, 4 feet

23-5-g 2 cables, 65 fathoms

23-5-h no

23-6-a 140 grains

23-6-b 12 scruples

23-6-c 240 grains

23-6-d 240 grains

23-6-e 2 scruples, 6 grains

23-6-f 118 drams, 1 scruple

24-7-a 10 gallons

24-7-b 45 pints, 3 gills

24-7-c 22 quarts, 1 pint

24-7-d 5 gallons, 2 quarts

24-7-e 5 gallons, 2 quarts, 1 pint, 3 gills

24-8-a 432 square inches

24-8-b 45 square feet

24-8-c 3 square yards, 1 square foot

25-1 true

25-2 false

25-3 false

25-4 true

25-5 false

25-6 questionable

25-7 false

25-8 questionable

25-9 true

25-10 true

25-11 true

27-1 smaller

27-2 8

27-3 more than 4

27-4 Canada

27-5-a 1 chain
b 10 inches
c 2 miles
d 23 chains

28-6-1 144

28-6-2 1728

28-6-a 2 boxes. 2 left

28-6-b 5 boxes. 8 left

28-6-c 1 box. 2 left

28-6-d 7 boxes. 0 left

28-7 no

30-a 20 elts

30-b 27 mouts

30-c 3 elts, 3 mouts

30-d 5 elts, 0 mouts

30-e 10 stots, 3 elts

31-a no

31-b yes

31-c no

31-d yes

31-e no

31-f yes

31-g no

31-h no

32-top part b

32-a 0 stots, 1 elt, 1 mout

32-b 0 stots, 2 elts, 0 mouts

32-c 1 stot, 0 elts, 0 mouts

32-d 1 stot, 1 elt, 1 mout

32-e 0 stots, 0 elts, 8 mouts

32-f 2 stots, 0 elts, 0 mouts

32-g 0 stots, 4 elts, 0 mouts

32-h 0 stots, 0 elts, 0 mouts

33-a 0 stots, 1 elt, 3 mouts

33-b 1 stot, 1 elt, 0 mouts

33-c 63 stots, 0 elts, 0 mouts

33-d 0 stots, 0 elts, 5 mouts

33-e 4 stots, 0 elts, 0 mouts

33-f 22 stots, 0 elts, 0 mouts

33-g 478 stots, 0 elts, 0 mouts

33-h 95 stots, 3 elts, 0 mouts

33-i 10 stots, 3 elts, 1 mout

35-a 3 stots, 0 elts, 0 mouts

35-b 1 stot, 3 elts, 0 mouts

35-c 0 stots, 1 elt, 0 mouts

35-d 2 stots, 1 elt, 0 mouts

35-e 11 stots, 1 elt, 0 mouts

38-top-a 4 yards, 0 feet, 0 inches

38-top-b 0 stots, 4 elts, 8 mouts

38-top-c 7 gallons, 0 quarts, 0 pints

38-top-d 0 gallons, 3 quarts, 0 pints

38-top-e 1 gallon, 1 quart, 0 pints

38-top-f 543 yards, 0 feet, 0 inches

38-top-g 181 yards, 0 feet, 0 inches

38-top-h 15 yards, 0 feet, 3 inches

38-bottom part a

39-a 108 inches

39-b 20 quarts

39-c 40 pints

39-d 10 pints

39-e 450 mouts

39-f 36 inches

39-g 3600 minutes

39-h 36000 seconds

39-i 1 stot

39-j yes

40-middle a,c,d

40-bottom part b

41-a 3 stots, 0 elts, 5 mouts

41-b 10 stots, 1 elt, 6 mouts

41-c 6 stots, 0 elts, 0 mouts

41-d 14 stots, 0 elts, 0 mouts

41-e 2 stots, 0 elts, 0 mouts

41-f 2 stots, 4 elts, 8 mouts

41-g 24 stots, 3 elts, 0 mouts

41-h 33 stots, 0 elts, 0 mouts

41-i 20 stots, 1 elt, 0 mouts

41-j 2 stots, 1 elt, 1 mout

42-a yes

42-b yes

43-middle b,d

43-bottom part b

44-a yes

44-b yes

45-a 25 elts

45-b 8 feet

45-c 128 inches

45-d 76 inches

45-e 94 mouts

45-f 13 pints

45-g 44 gills

45-h 125 minutes

45-i 225 mouts

45-j 180 mouts

45-bottom 14, 16, 17, 20, 39

48-a 72 inches

48-b 5280 feet

48-c 2000 acres

48-d 120 grains

48-e 128 pints

48-f 1000000 millimeters

48-g 5 feet

48-h 84 inches

48-i 5103 links

48-j 315 scruples

48-k 90 gills

48-l 6000 feet

48-m 3200 links

48-n 1296 sq. inches

48-o 46656 cubic inches

49-a 4 yards

49-b 1 square foot

49-c 3 hectares, 27 acres

49-d 4 bushels, 3 pecks

49-e 3 nautical miles, 5 cable lengths, 46 fathoms

49-f 2 yards, 3 inches

49-g 3 gallons, 1 quart, 2 gills

49-h 1grain, 1 scruple, 5 drams

49-i 6 furlongs, 6 chains

49-j 2 cubic inches, 2 cubic feet

49-k 2 nautical miles

49-l 1 kilometer

49-m 1 kilometer, 2 hectometers, 5 decameters, 3 meters 6 decimeters, 9 centimeters 4 millimeters

49-n 6 meters, 6 decameters, 2 hectometers, 1 kilometer

49-o 1 hectare

51-a false

51-b false

51-c true

51-d depends on how you look at it

51-e true

59-1 5 yards

59-2 1 foot, 2 inches

59-3 14 links, 166 chains

59-4 38 fathoms

59-5 5 scruples, 19 grains

59-6 4 yards, 1 foot, 9 inches

59-7 159 tons, 530 pounds

59-8 3 gills

59-9 2 bushels, 3 pecks, 5 quarts

59-10 5 miles, 6 furlongs

59-11 19 square yards, 5 square feet, 143 square inches

59-12 46 cubic yards, 12 cubic feet, 1726 cubic inches

59-13 2 pounds

59-14 80 yards, 1 foot, 11 inches

70-1 14 feet, 6 inches

70-2 10 inches

70-3 22 bushels, 1 peck, 4 quarts

70-4 4 square yards, 6/25 square feet

70-5 1000 pounds, 8 1/2 ounces

70-6 50 1/2 centiares

70-7 8 pounds, 4 ounces, 20 grains

70-8 16 yards, 9 2/5 inches

70-9-a yes

70-9-b no

72-a yards

72-b 1.09 yards

72-c 2.18 yards

72-d multiply by 1.09

72-e 1 meter

72-f 3 meters

72-g divide by 1.09

75-1 11.9 meters

75-2 1.83 meters

75-3 168 cubic inches

75-4 5.95 pints

75-5 438.6 square centimeters

75-6 2 meters, 4 decimeters, 8 centimeters, 9.2 millimeters

(any form which is the same should be accepted)

75-7 4.3 pints (dry measure)

NOTE TO TEACHERS

This has been one of our most successful units. Those teachers who have used it have experienced a wide variety of reactions from their students, nearly all of which have been favorable. The unit has been used with seventh, eighth, and ninth graders with seemingly equal effectiveness.

The one area of greatest concern at the present time is whether it is best to read the story to the students or let them read it themselves. It has been our experience that a more effective job of teaching occurs when the story is read to the students, particularly if they are low achievers. If you choose to read the story to your students, we would anticipate that you would use thermofax spirit masters of the problems and exercises as they appear in this unit.

We are presently contemplating putting the story on tape and handing out the problems on worksheets to the students. We would be most appreciative if you would forward any comments you might have after using this unit to:

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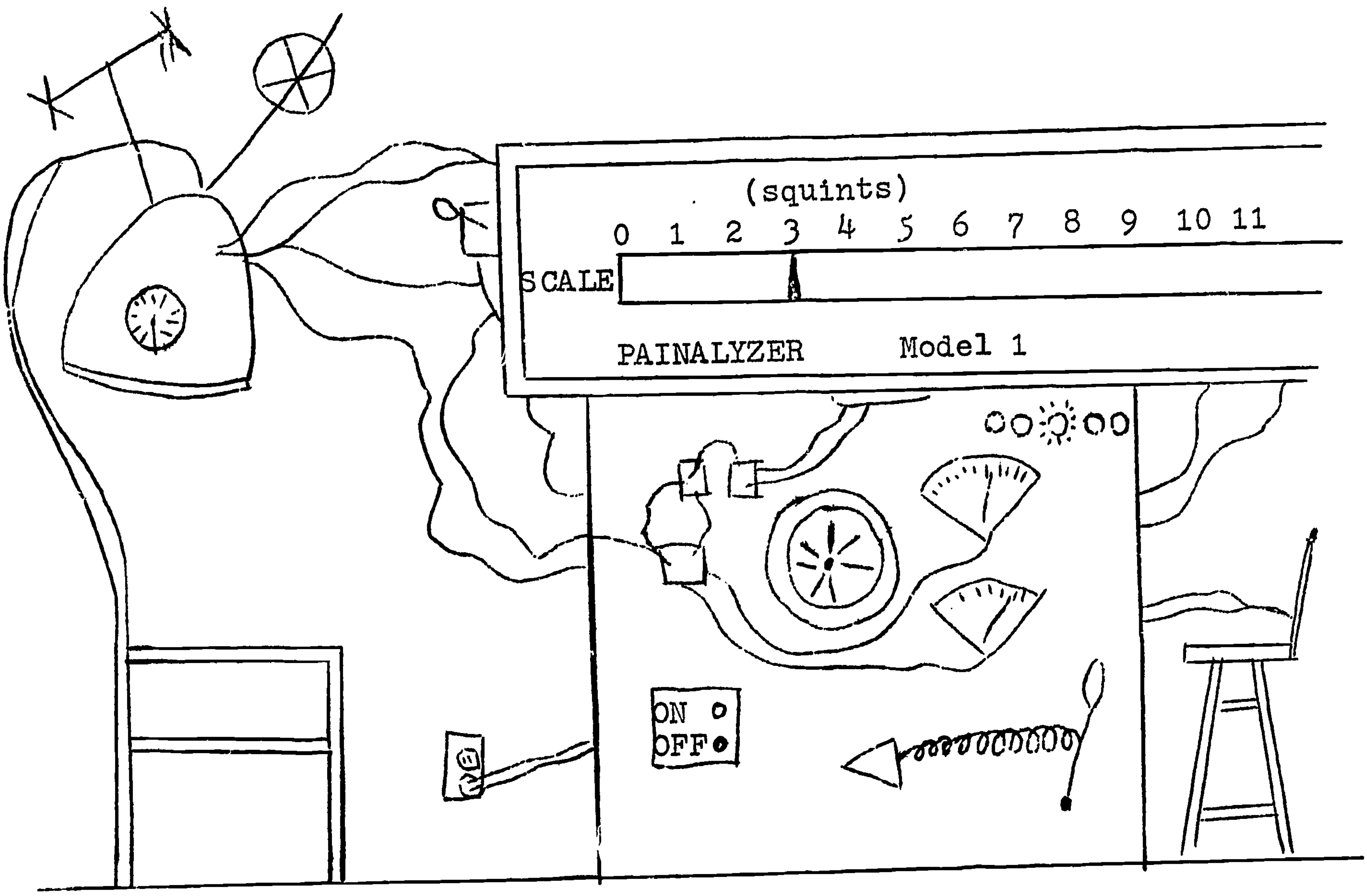
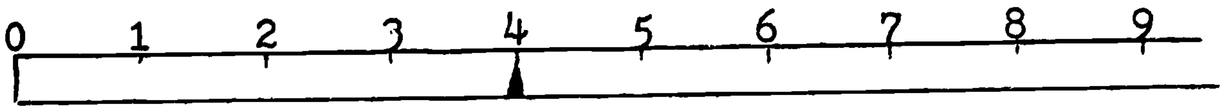
Chapter ONE
(Measurement using two units within a system)

PART I

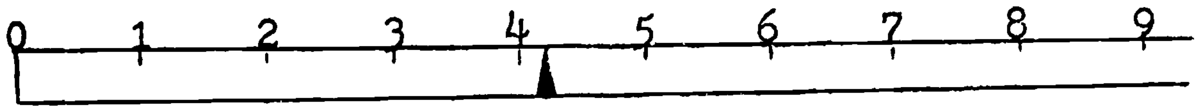
In the year 2000 A.D., Doctor Seymore Squint announced the invention of a marvelous machine that measured headache pain. The machine was called a Painalyzer. Doctors soon began learning all kinds of interesting things about their patients. Sometimes people who complained the most didn't have much pain at all. Others who hardly said a word had dreadful headaches. (Dr. Squint explained that some of these people didn't complain because it hurt too much when they talked. Some of them, he explained, were born with terrible headaches, and they very likely thought it was normal to have strange feelings in their heads.)

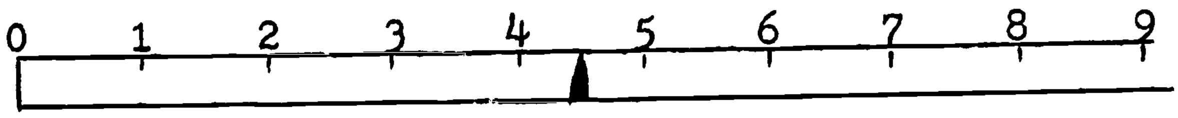


The first machine measured headaches in <u>squints</u>. (Dr. Squint named the unit of measure after himself.) The control box on the machine had a long row of numbers on it. When a sufferer sat under the special helmet, an arrow quickly pointed to the correct number.



It didn't always point exactly to some number. After all, it was certainly possible to have a headache which was a little more than 4 squints and not quite as bad as 5 squints.





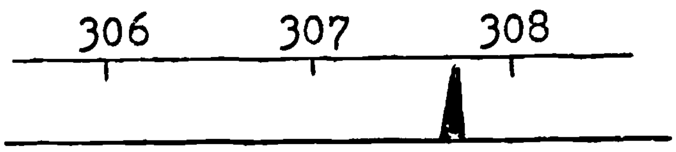
In such cases, a doctor would just glance at the scale and write down the number he thought was nearest the arrow. Since the squint was a rather small amount of pain, the doctor's guesses were usually close enough.

These numbers were then used by the nurse, who called the druggist, who then knew how much medication to give for the headache.

It didn't matter much if the doctor said the patient had 4 squints of pain when he actually had about 4½ squints of pain. The squint was such a small amount of pain that some doctors claimed that less than a squint of pain was more like a pleasant tickle than like a headache.

If you were the doctor, what number would you write down for each of the following cases?

1)

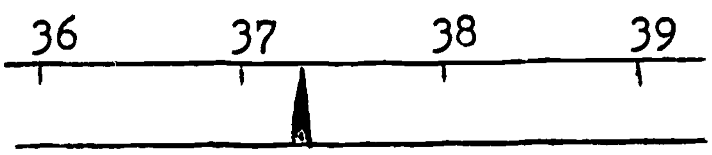


Name: Mrs. Dragonpaw

Notes: very upset...just smashed their new car

_____squints

2)

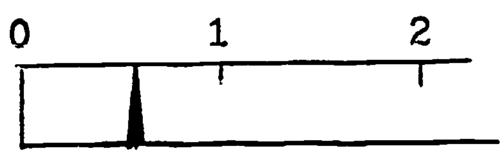


Name: "Pole" Dunker

Notes: thinks he stepped on soap bar in shower.. ...doesn't remember for sure

_____squints

3)

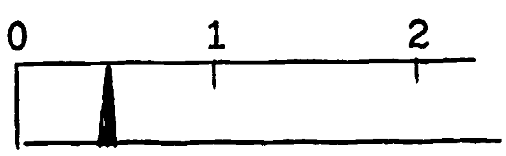


Name: Mr. Chond

Notes: had nothing better to do....came here.. ..thought he might have a headache

_____squints

4)

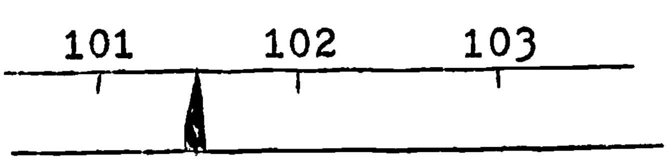


Name: Mrs. Chond

Notes: same...thought she might have a headache. ...couldn't be healthier

_____squints

5)



Name: Rick Featherton

Notes: too much TV...every single late, late movie for 3 weeks.

_____squints

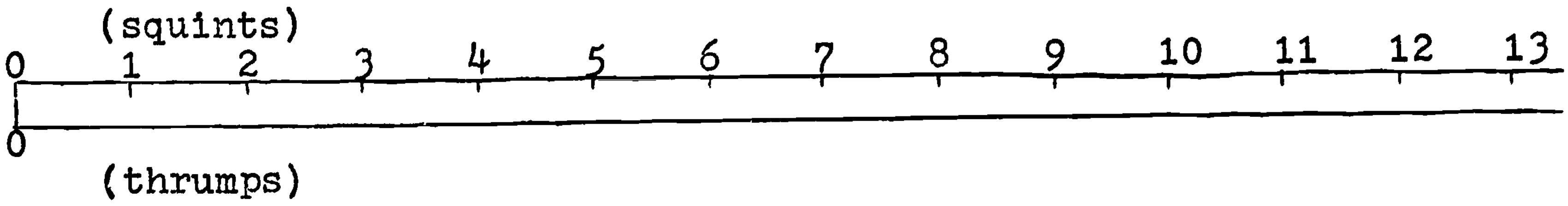
As it usually happens with measuring things, not everyone was happy with the size of the units. Before long the druggists began to complain. Their headache tablet (the thrumprin) was strong enough to remove exactly 6 squints of pain. The druggists said that the squint was too small, and before long the druggists demanded that a new unit be used for headaches. The new unit of pain was called the "thrump," where one thrump was exactly the same as 6 squints. (1 thrump = 6 squints)

This caused problems for the doctors. They still liked squints better. They didn't have time to multiply or divide or add or whatever you have to do to change squints to thrumps. They didn't want to build different machines either. Somehow they had to do something to the scales on their old machines so that it would give the pain either in squints or thrumps.

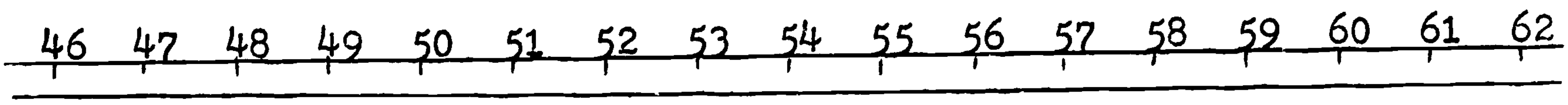
One doctor suggested that they could sharpen the bottom of the pointer on the scale and write the numbers for thrumps below the pointer. This way they could read the top numbers if they wanted the pain in squints, and they could read the bottom numbers if they wanted the pain in thrumps.

Since the doctors were doctors and not engineers, they sent for a representative from the Painalyzer Company, whose job it was to put the new set of numbers on the machine and sharpen the bottom of the pointer.

6) Where should the Painalyzer man put numbers below for thrumps? ______________________



7) Where should the numbers go below the following part of the scale? ______________________



8) Which numbers on the squint scale will have thrump numbers below them? (Check the statement which seems best.)

a.___numbers with a 6 in the unit's place

b.___numbers which are even numbers

c.___numbers which can be divided evenly by 6

d.___numbers which are 6 bigger than some other number

9) On the scales in the questions above, write down the correct thrump numbers in their places.

10) After you have chosen a squint number which will have a thrump number beneath it, which of the following rules seems best for finding the correct number to write down?

a.___subtract 6 from the squint number

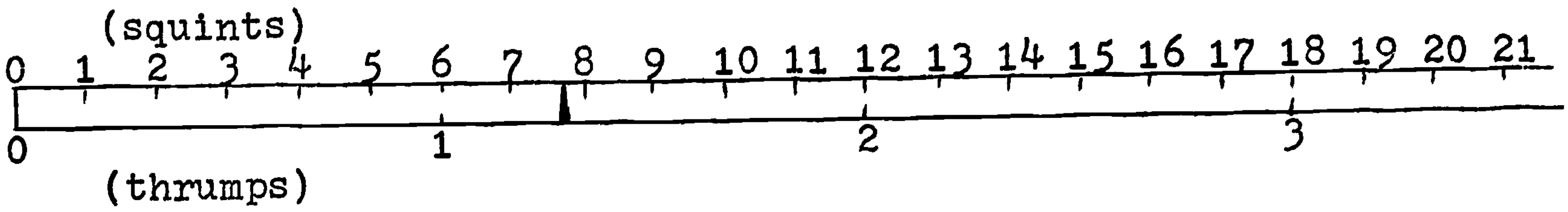
b.___divide the squint number by 6 - use the quotient for thrumps

c.___divide the squint number by 6 - use the remainder for thrumps

d.___add the digits of the squint number

With squints and thrumps the nurse's job became much more difficult. When the doctor told her the pain in squints, she had to call the druggist and describe the pain in thrumps. When the druggist gave her a number in thrumps, she had to give the doctor the correct number in squints.

NOTE: An equal sign with a dot above it means that the two numbers or quantities are "about the same." For example, 17 squints is more than 2 thrumps and less than 3 thrumps. But 17 squints is closer to 3 thrumps than it is to 2 thrumps, so the nurse would write:
17 squints $\doteq$ 3 thrumps



11) Using the scale above, write down the numbers that you think the nurse should use.

a. 5 squints $\doteq$ ____ thrumps

b. 3 thrumps $\doteq$ ____ squints

c. ____ thrumps $\doteq$ 18 squints

d. ____ squints $\doteq$ 2 thrumps

e. ____ thrumps $\doteq$ 2 squints

f. 15 squints $\doteq$ ____ thrumps

12) In which of the examples above could the dots be erased from the equal signs? ______________________

NOTE: If you were asked to change 9 squints to thrumps, you would notice that 9 squints is exactly as close to 1 thrump as it is to 2 thrumps. In such cases we usually pick the largest number. (9 squints $\doteq$ 2 thrumps)

13) In which of the examples in problem 11 does this note help you decide which number to use? ________________

13) Which rule sounds the best for changing thrumps to squints?

a.___divide the number of thrumps by 6 to get squints

b.___multiply the number of thrumps by 6 to get squints

c.___add 6 to the number of thrumps to get squints

d.___subtract 6 from the number of thrumps to get squints

14) For changing squints to thrumps, which rule sounds the best for getting a fairly close answer?

a.___divide the number of squints by 6 -- the remainder is close to the best answer

b.___multiply the number of squints by 6 to get the number of thrumps

c.___divide the number of squints by 6 -- the quotient is close to the best answer

15) Using the rules you chose in parts "13" and "14," complete the following list of conversions. (Where you can, you may check your work using the scale on the page before this one.)

a. 12 squints ≑ _____thrumps

b. 15 squints ≑ _____thrumps

c. 2 thrumps ≑ _____squints

D. 146 squints ≑ _____thrumps

e. 30 thrumps ≑ _____squints

f. 11 squints ≑ _____thrumps

g. 13 squints ≑ _____thrumps

16) Does the rule you chose in part "14" always give the best possible answer? _______yes, _______no

(If you said "no," give at least one example where the rule does not give the best possible answer.)

SUMMARY
(part I)

I. 1 thrump = 6 squints

II. The only squint numbers with thrump numbers below them are squint numbers which can be divided evenly by 6.

1 thrump = 6 squints
2 thrumps = 12 squints
3 thrumps = 18 squints
4 thrumps = 24 squints
etc.

III. To change thrumps to squints, simply multiply the number of thrumps by 6.

IV. To change squints to thrumps, divide the number of squints by 6. If the remainder is less than 3, the quotient is the best answer.

If the remainder is 3 or more, add 1 to the quotient to get the best answer.

PART II

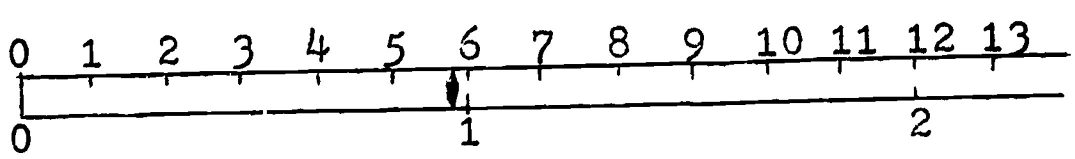
In the year 2005 A.D., a new headache pill was developed. There was already a pill called the thrumprin. (A thrumprin was exactly strong enough to relieve a sufferer of one thrump of headache pain.) The new pill was the squin, and a squin was exactly strong enough to relieve a sufferer of one squint of pain.

Although the two wonder-drugs were truly wonderful, they did cause problems. A thrumprin cost more than 5 squins, but 6 squins cost more than a thrumprin. Naturally, the patients wanted to spend as little money as possible to get rid of their headaches.

The job of finding the least expensive way to get rid of a headache was often given to the nurses.

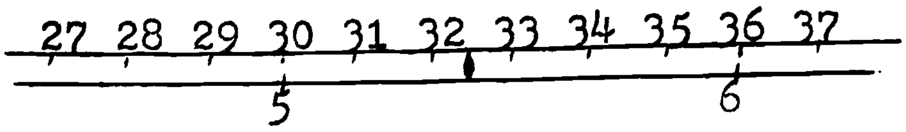
- - - - - - - - - - - - - - - - - - - -

In which of the following cases did the nurse make the best suggestion possible? (To put it another way, in which of the following cases did the patient get rid of his headache for the least amount of money possible?)



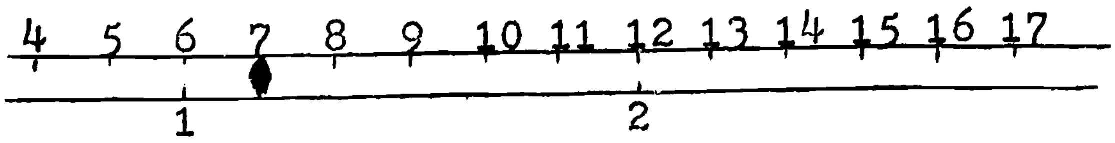
Prescription: __0__ thrumprins, __6__ squins

Was this the least expensive prescription? ____________

2) 

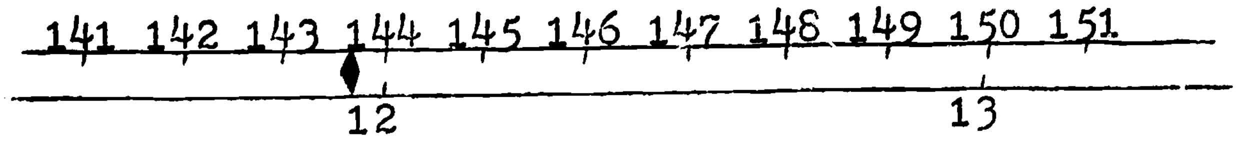
prescription: __4__ thrunprins, __8__ squins

Was this the least expensive prescription? ____________

3) 

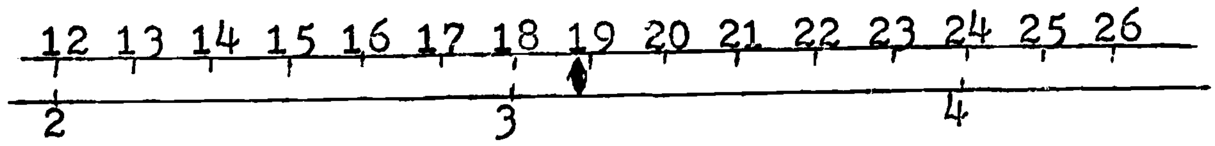
prescription: __1__ thrunprins, __1__ squins

Was this the least expensive prescription? ____________

4) 

prescription: __12__ thrumprins, __0__ squins

Was this the least expensive prescription? ____________

5) 

prescription: __0__ thrunprins, __19__ squins

Was this the least expensive prescription? ____________

6) In "1" through "5" write down a better answer if you are not satisfied with the answer given. If you are satisfied, simply write "OK."

1) ____________________ 4) ____________________

2) ____________________ 5) ____________________

3) ____________________

7) If an answer is the best possible answer (the least expensive prescription), then the most squins the patient might have to buy is ___3, ___4, ___5, ___6, ___7.
(check one)

8) If the doctor gave the nurse a number in squints, what would the nurse do if she couldn't find her scale for changing squints to thrumps and squints so that the prescription would be the least expensive?

a.___leave the number alone -- prescribe that number of squins

b.___multiply the number by 6 -- prescribe that number of squins

c.___divide the number of squints by 6 -- the quotient is the number of thrumprins and the remainder is the number of squins.

d.___divide the number by 6 -- the remainder is the number of thrumprins and the quotient is the number of squins

\- - - - - - - - - - - - - -

NOTE: Perhaps without knowing it, in problem "8" the nurse was faced with putting measurements in "<u>LOWEST DENOMINATION</u>." This means writing the numbers using both units, thrumps and squints, so that <u>the number of thrumps is as large as possible and the number of squints is as small as possible</u>.

If the nurse was lucky, she discovered that the best method was method "c" in exercise "8."

EXAMPLE: change 44 squints to lowest denomination

$$\begin{array}{r} 7 \\ 6\overline{)44} \\ \underline{-42} \\ 2 \end{array} \quad -42 = 6 \times 7$$

Looking at the division, we see that 44 is the same as 42 + 2.

We also know that whenever we have 6 squints we have 1 thrump.

Putting things together, we see that:

$$\begin{aligned} 44 \text{ squints} &= (42 + 2) \text{ squints} \\ &= 42 \text{ squints} + 2 \text{ squints} \\ &= 7 \times 6 \text{ squints} + 2 \text{ squints} \\ &= 7 \text{ thrumps} + 2 \text{ squints} \\ &= \underline{\ 7\ } \text{ thrumps, } \underline{\ 2\ } \text{ squints} \end{aligned}$$

9) Change each of the following to lowest denomination.

a. 14 squints = _____thrumps, _____squints

b. 48 squints = _____thrumps, _____squints

c. 5 squints = _____thrumps, _____squints

d. 666 squints = _____thrumps, _____squints

e. 50 squints = _____thrumps, _____squints

f. 6 squints = _____thrumps, _____squints

g. 4 squints = _____thrumps, _____squints

h. 669 squints = _____thrumps, _____squints

10) In which parts of problem "9" was there nothing to do?

____

11) In problem "9," which rule would tell you that there is nothing to do?

A.____if the number is less than 12 there is nothing to do.

B.____if the number of squints is divisible by 6 there is nothing to do.

C.____if the number of squints is less than 6 there is nothing to do.

NOTE: Whenever two different sized units are used to measure the same quantity, they are related by a number
For some examples:

1. inches and feet are related by "12"
(1 foot = 12 inches)
2. pounds and ounces are related by "16"
(1 pound = 16 ounces)
3. squints and thrumps are related by "6"
(1 thrump = 6 squints)
4. spans and fathoms are related by "8"
(1 fathom = 8 spans)

These numbers are called "CONVERSION NUMBERS" or sometimes they are called "Conversion Factors."

SUMMARY
(part II)

I.. 1 thrump = 6 squints

II. The only squint numbers with thrump numbers below them are squint numbers which can be divided evenly by 6.

III. To change thrumps to squints, multiply the number of thrumps by 6.

IV. To change squints to thrumps, divide the number of squints by 6.

If the remainder is less than 3, the quotient is the best answer.

If the remainder is 3 or more, add 1 to the quotient to get the best answer.

V. To change a squint number to lowest denomination divide the number by 6.

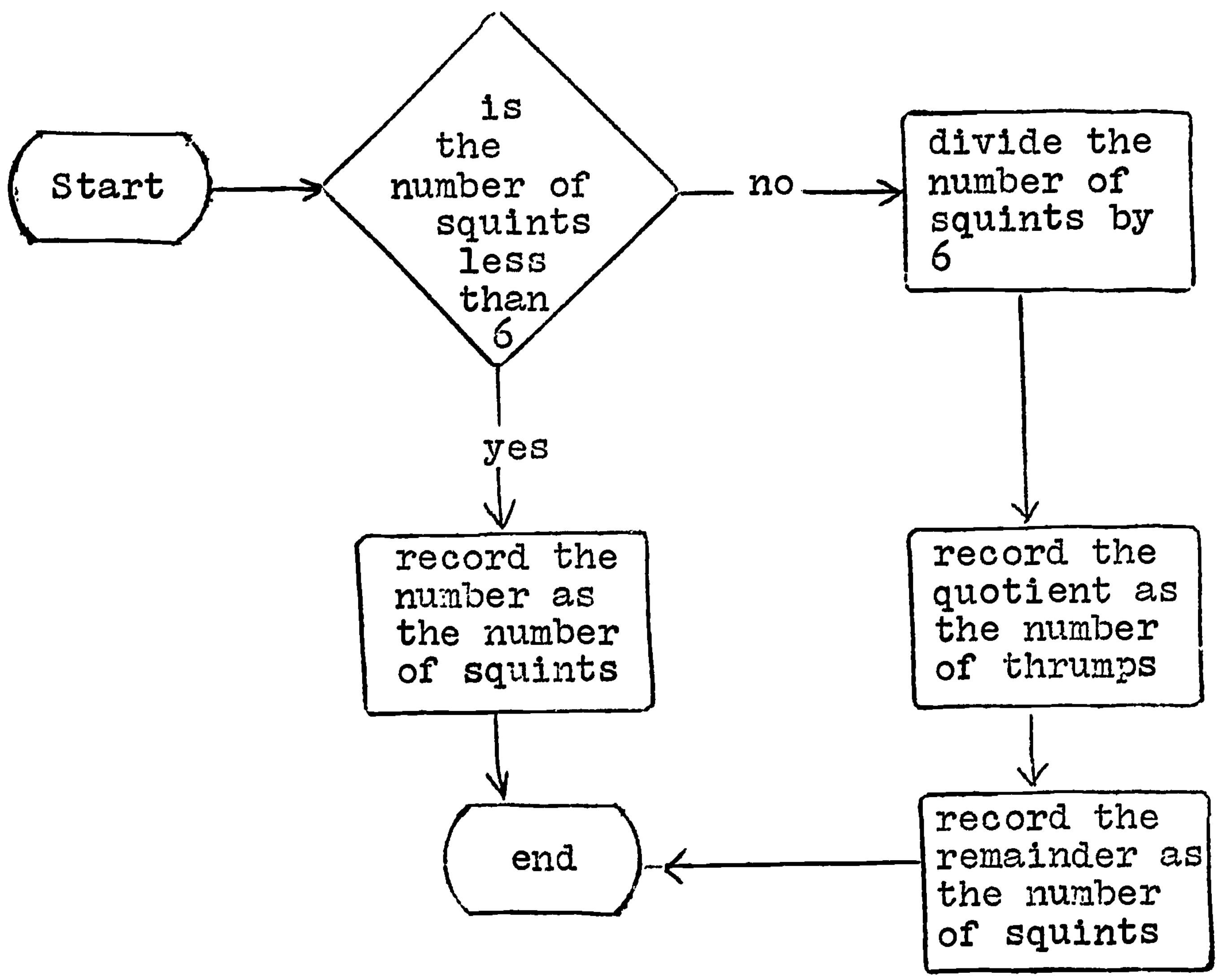
The quotient is the number of thrumps.

The remainder is the number of squints.

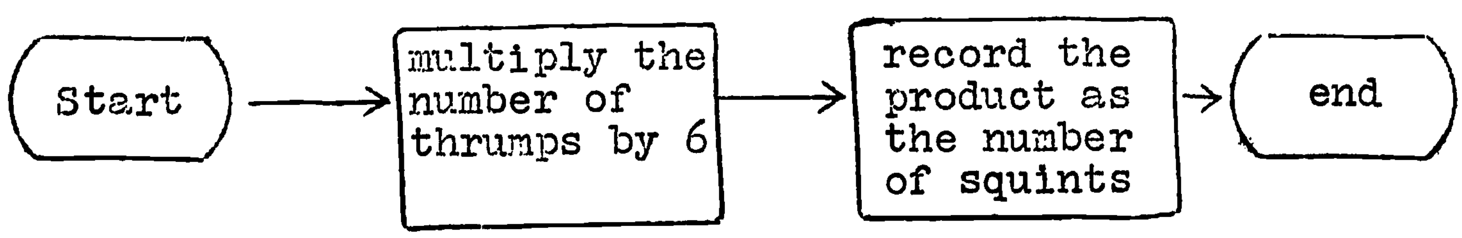
VI. A number which relates two different-sized units which measure the same type of quantity is called a "conversion number" or "conversion factor."

PART III

If the nurses were asked to construct flow-charts for changing a squint number to lowest denomination, the flow-chart might look something like the following:



A flow-chart for changing thrumps to squints would be even shorter:



Using the flow-charts, change each of the following readings to lowest denomination:

7 squints = ____________________

8 thrumps = ____________________

60 squints = ____________________

61 squints = ____________________

62 squints = ____________________

1 thrump = ____________________

1 squint = ____________________

600 squints = ____________________

700 squints = ____________________

It could have happened that when the thrumprin was developed it was only strong enough to remove 5 squints of pain instead of 6. In that case the squint would remain the same, but the druggists would have insisted that 1 thrump = 5 squints.

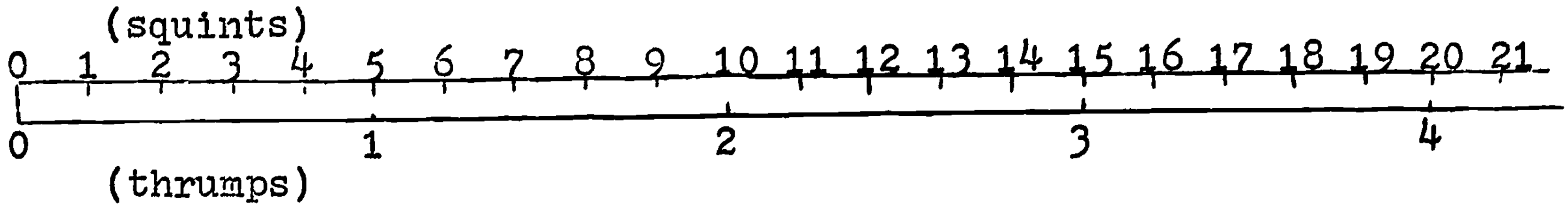
In that case, where would thrump numbers have to be put on the following parts of the Painalyzer's scale? (Put in the proper numbers where you think they should be.)

(squints)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

(thrumps)

47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68



On this page pretend that 1 thrump = 5 squints.

What should the nurses write down in the following cases if they were changing thrumps to squints? (You may use the scale at the top of the page to help you decide.)

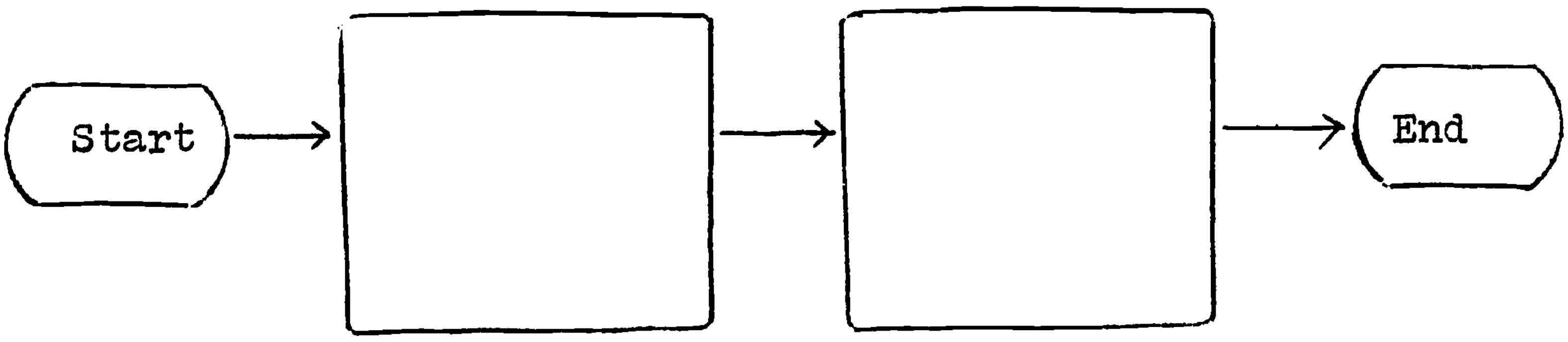
1 thrump = ________________squints

5 thrumps = ________________squints

3 thrumps = ________________squints

6 thrumps = ________________squints

How would you fill in a flow-chart for changing thrumps to squints? (Look at the flow-chart on page 14 if you think it will be helpful.)



See if you can write the following numbers in lowest denomination. (Remember, 1 thrump = 5 squints.)

7 squints = _____thrumps, _____squints

6 squints = _____thrumps, _____squints

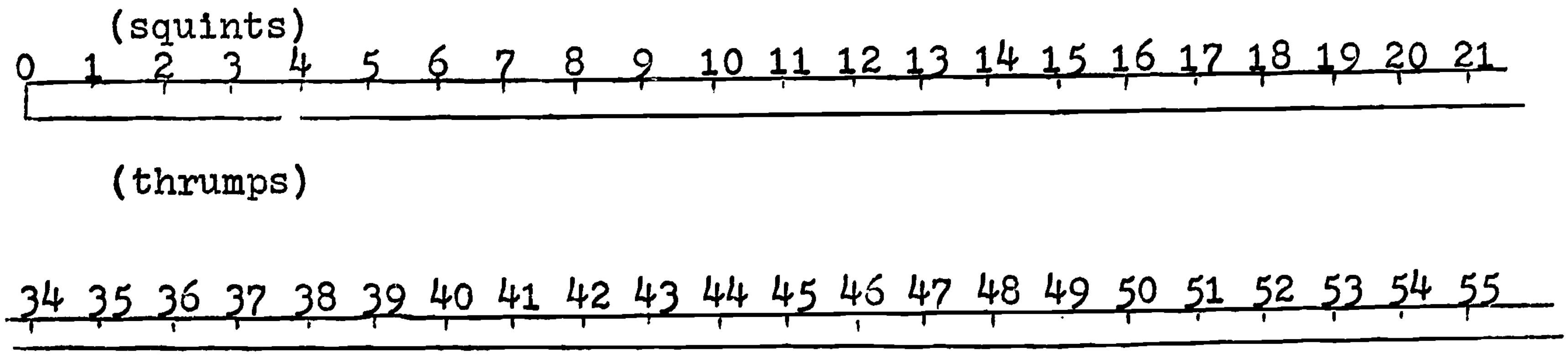
23 squints = _____thrumps, _____squints

50 squints = _____thrumps, _____squints

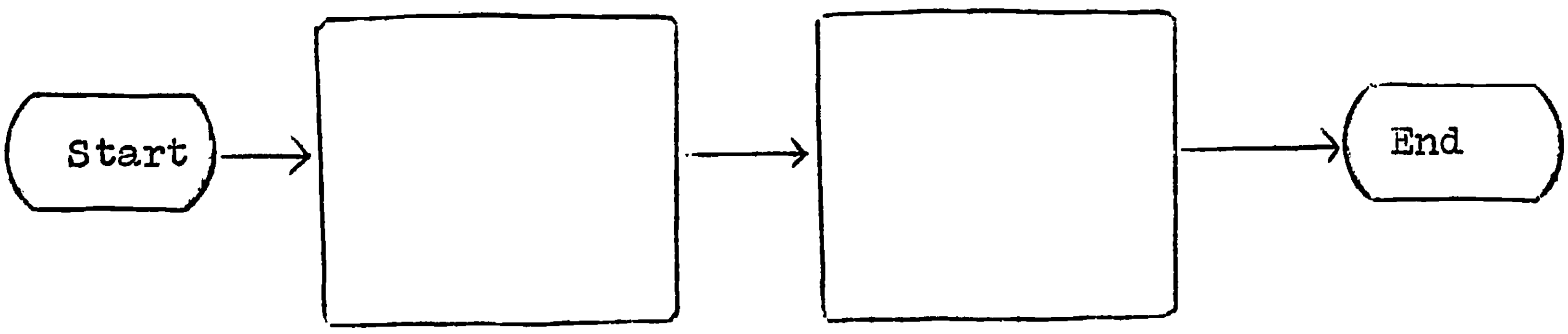
251 squints = _____thrumps, _____squints

We've just finished pretending that 1 thrumprin was only strong enough to remove 5 squints of pain instead of 6. On this page, let's pretend that the thrumprin was so powerful that it removed exactly 12 squints of pain. In that case, to keep the druggists happy, we should make 1 thrump = ______squints.

Where should thrump numbers go on the following parts of the scale? ____



In this case, how would the flow-chart for changing thrumps to squints look?



What would you write in each of the following cases?

1 thrump = _______squints

12 thrumps = _______squints

2 thrumps = _______squints

10 thrumps = _______squints

(squints)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

0 1

(thrumps)

22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43

2 3

44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65

4 5

On this page pretend that <u>1 thrump = 12 squints</u>

What would each of the following be if reduced to lowest denomination?

13 squints = _____thrumps, _____squints

24 squints = _____thrumps, _____squints

44 thrumps = _____thrumps, _____squints

47 squints = _____thrumps, _____squints

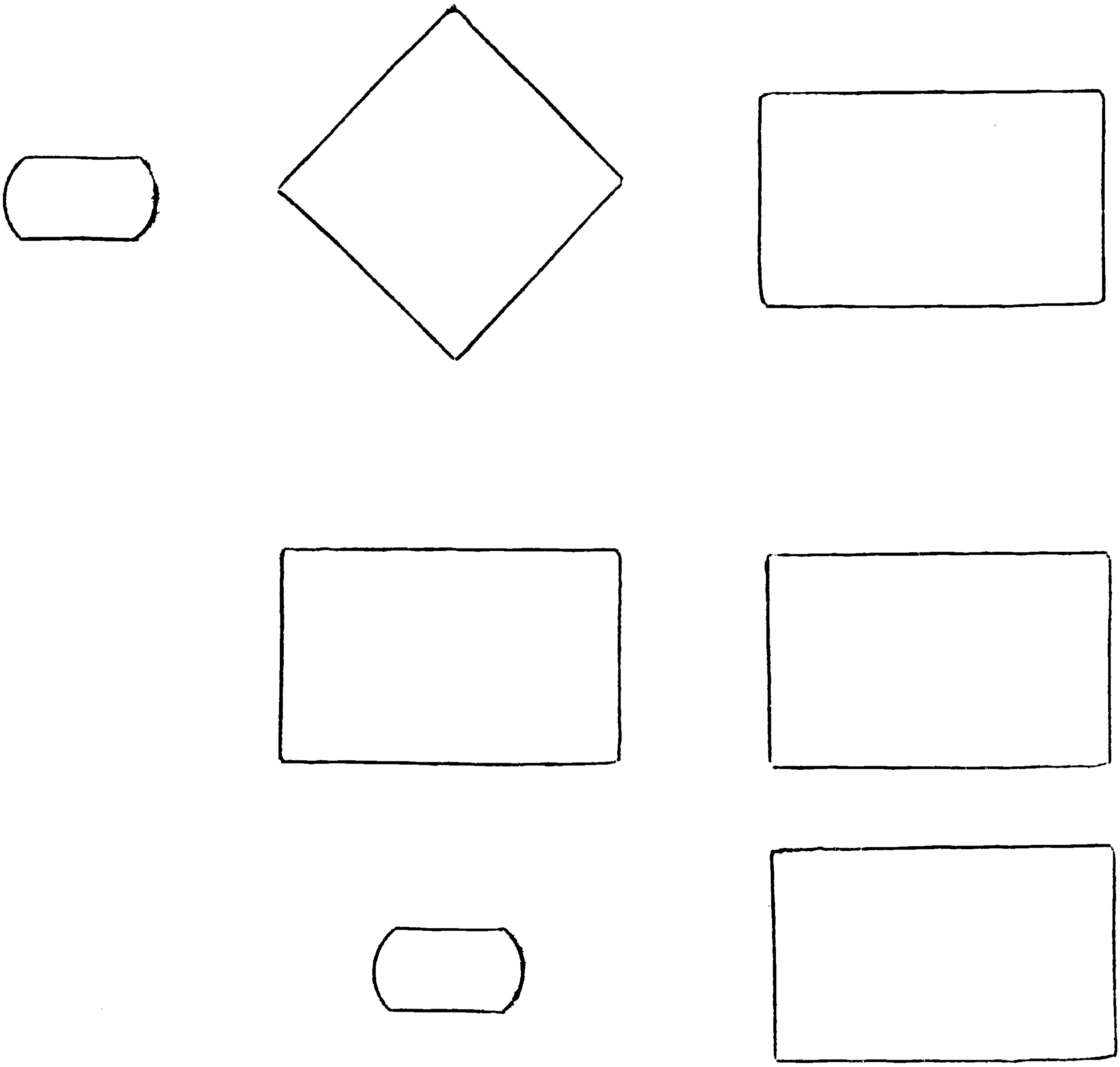
48 squints = _____thrumps, _____squints

49 squints = _____thrumps, _____squints

50 squints = _____thrumps, _____squints

Still pretending that 1 thrump = 12 squints, how would the flow-chart for changing squints to lowest denomination look?

(Work on the next page - -
you may use the flow-chart on
page 14 if you think it
will help.
You may also use the scale
on the top of this page.)



Using only the flow-chart you have made, try to write each of the following in lowest denomination.

14 squints = _____thrumps, _____squints

5 squints = _____thrumps, _____squints

120 squints = _____thrumps, _____squints

121 squints = _____thrumps, _____squints

122 squints = _____thrumps, _____squints

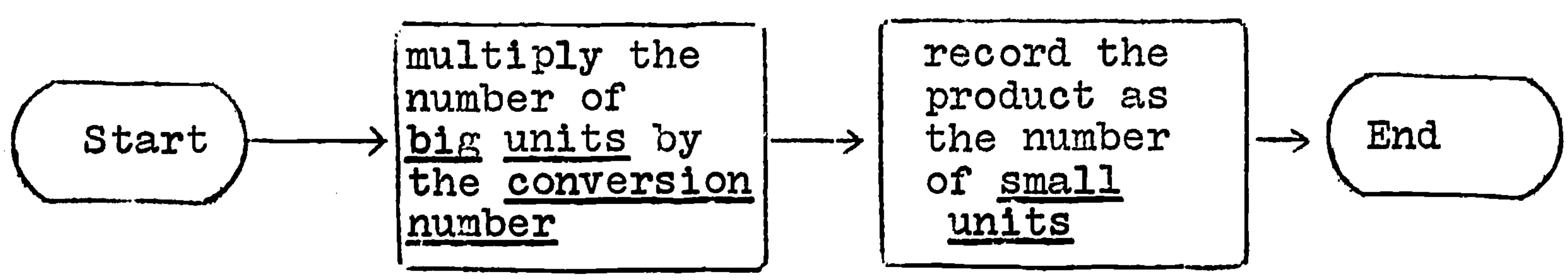
PART IV

In part III it didn't seem to make much difference how many squints there were in a thrump - - the flow-charts all looked pretty much the same.

In each case there were three things to consider:

a small unit	(the squint)
a big unit	(the thrump)
a conversion number	(we tried 6, 5, and 12)

If we were to make a flow-chart that would tell us what to do, no matter which conversion number we used, the flow-chart would probably look like the following one if we were changing big units to small units.



If 1 thrump = 6 squints, we would

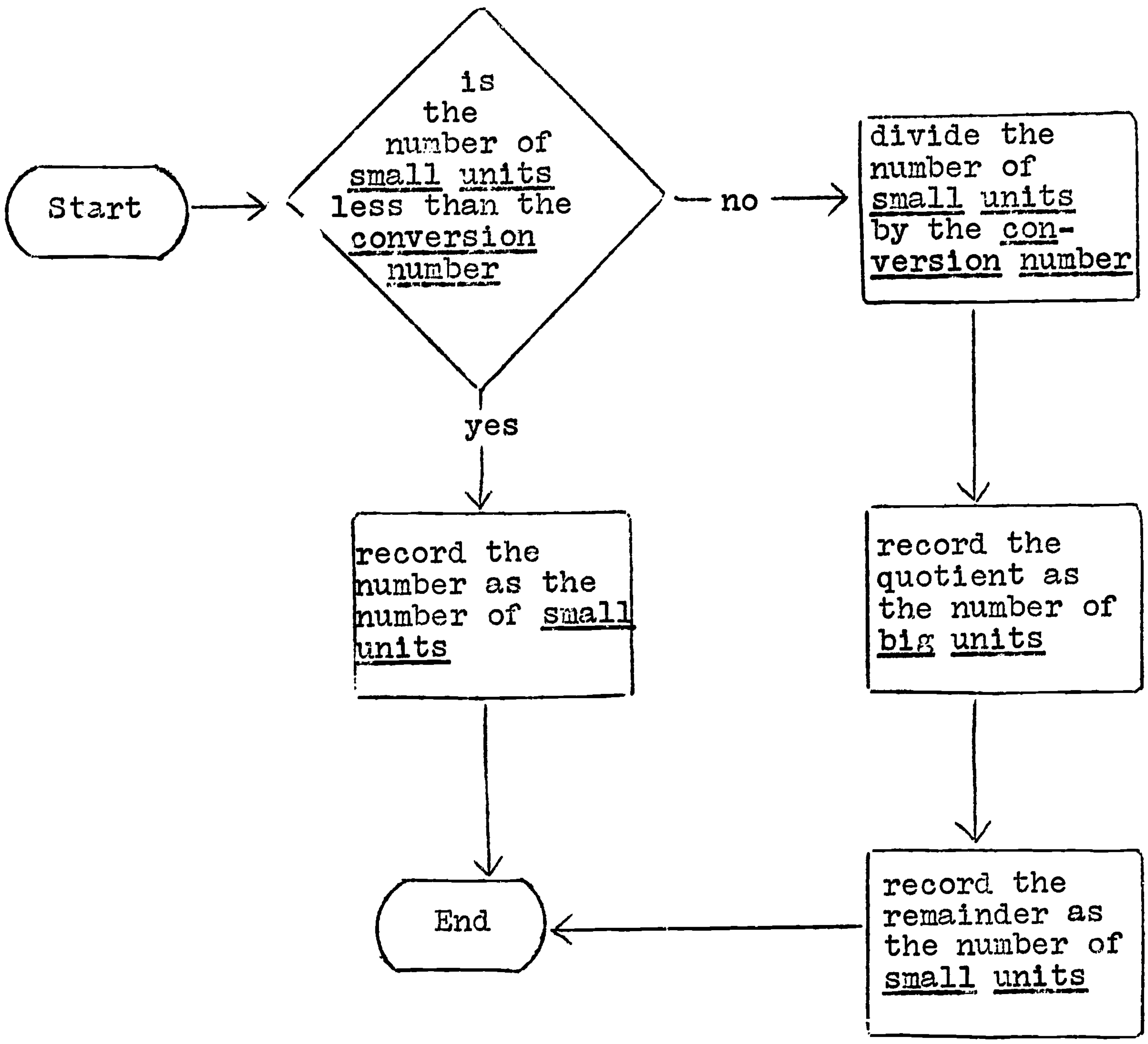
replace big units by ______________________

replace small units by ______________________

replace conversion number by ________________

(Compare the flow-chart above to the one on page 14, the one on page 16, and the one on page 17.)

A flow-chart for changing small units to lowest denomination would very likely look like this:



(Compare this flow-chart to the one on page 14 and the one on page 19.)

PART V

EXERCISES

(Until you become confident, you should use the flow-charts on pages 20 and 21.)

1. In the English System for measuring length:
   12 inches = 1 foot
   3 feet = 1 yard
   1760 yards = 1 mile

   (a) Change 4 feet to inches.

   (b) Change 5 yards to feet.

   (c) Change 2 miles to yards.

   (d) Change 56 inches to lowest denomination using feet and inches, not yards.

   (e) Change 11 feet to lowest denomination using yards and feet.

   (f) Change 2000 yards to lowest denomination using miles and yards.

2. Using Surveyor's Measure for measuring length:
   10 chains = 1 furlong

   (a) Change 5 furlongs to chains.

   (b) Change 72 chains to lowest denomination using chains and furlongs.

   (c) Change 100 chains to lowest denomination using chains and furlongs.

3. Again, using Surveyor's Measure for measuring length:
   100 links = 1 chain

   (a) Change 4 chains to links.

   (b) Change 357 chains to lowest denomination using links and chains.

4. CLASS DISCUSSION It turns out that 1 link is about 7.92 inches. How long is a furlong, expressed in the English System for measuring length?

5. Using Nautical Measure (used at sea) for measuring length:
6 feet = 1 fathom
100 fathoms = 1 cable length

(a) Change 5 fathoms to feet.

(b) Change $\frac{1}{2}$ fathom to feet.

(c) Change 3 cables to fathoms.

(d) Change 300 fathoms to feet.

(e) By looking only at work you already did, how many feet are there in 3 cables?

(f) Change 28 feet to lowest denomination using feet and fathoms.

(g) Change 265 fathoms to lowest dencmination using fathoms and cables.

(h) CLASS DISCUSSION In addition to what was given above, 10 cables = 1 nautical mile.

If we think of a "foot" as we're used to seeing, namely the length of an ordinary ruler, when we're told that a nautical mile is 6,076 feet.

Is the "foot" used in the English System (our ruler) the same as the "foot" used by Nautical Measure?

6. Another system of measure that many of us are not acquainted with is the Apothecaries' Weight System. In this system:
20 grains = 1 scruple
3 scruples = 1 dram

(a) Change 7 scruples to grains.

(b) Change 4 drams to scruples.

(c) Change 12 scruples to grains.

(d) Looking at work you have already done, how many grains are there in 4 drams?

(e) Change 46 grains to lowest denomination using grains and scruples.

(f) Change 355 scruples to lowest denomination using drams and scruples.

7. The Liquid Measure System we use is the following:
4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

Pints, quarts, and gallons are perhaps known to all of you. Gills are not used very often. Chances are that if you asked for 320 gills of gasoline at a service station, they wouldn't have the slightest notion what you were talking about.

(a) CLASS DISCUSSION Just how many gallons are there in 320 gills?

(b) Change 183 gills to lowest denomination using gills and pints.

(c) Change 45 pints to lowest denomination using pints and quarts.

(d) Change 22 quarts to lowest denomination using quarts and gallons.

(e) Looking at parts (b), (c), and (d) above, write 183 gills as a number of gallons, quarts, pints, and gills.

8. We usually measure area by square inches, square feet, and square yards. It turns out that:
144 square inches = 1 square foot
9 square feet = 1 square yard

(a) Change 3 square feet to square inches.

(b) Change 5 square yards to square feet.

(c) Change 28 square feet to lowest denomination using square feet and square yards.

9. CLASS PROJECT Using the dictionary or an encyclopedia, look for peculiar systems of measurement. List the following about each system:

(a) How does the smallest unit compare to inches, feet, pounds, pints, square feet, or some other quantity everyone is familiar with?

(b) What are the different units in the system, and how are they related to each other?

(c) Who might use the system in his work?

Chapter TWO (Class Discussion Topics)

Part I

If you agree with the statement, write "true;" if you disagree, write "false;" and if you think it could be argued either way, put down a question mark.

1. Jake Shade owns a hardware store and he sells fencing by the foot. If he wants to make more money than he does now, he will make his ruler slightly shorter than the rulers used in school. ______

2. Since scales are very expensive pieces of equipment, butchers and packing houses could save a lot of bother if they first measured the length of a steer and then paid the farmer by a fixed price per inch. ______

3. There's really no good reason why carpenters and dress-makers don't just make their own rulers instead of paying for fancy rulers from a store. ______

4. When construction companies buy sand, they buy it by the ton. It would be too much fuss to buy and sell sand by the ounce. ______

5. Talking about ounces, pounds, and tons makes life too complicated. What people should do is decide to use just one method of weighing things, say pounds, and then never mention ounces and tons again. ______

6. Money doesn't really measure anything. ______

7. To check the accuracy of a ruler in your classroom, you simply measure it with another ruler in your classroom. ________

8. There isn't such a thing as a perfectly accurate ruler. ________

9. Some rulers are more accurate than other rulers. ________

10. If you were the king of a new isolated country, which had no system of measuring length, you could choose any length you wanted for the country's rulers. ________

11. If you were the king of the country in statement "10," it might be wise to choose one ruler as the "official ruler" so that when people began arguments about their own rulers, they could settle things by checking with the official ruler. ________

Part II (Class Discussion)

1. When the school nurse asked Seymore how much he weighed, he gave his answer in ounces. If he had given his answer in pounds, the number of units would have been . . . .

. . ___greater, ___smaller. (check the answer you choose)

2. Larry had 94¢ in his pocket. What is the smallest number of coins he could have had in his pocket?

____7, ____8, ____9

3. If there are 4 gills in a pint and 2 pints in a quart, how many gills are there in a quart?

_____more than 4, _____fewer than 4, _____4

4. 1.201 United States gallons = 1 Canadian gallon. If gasoline was priced at 35¢ per gallon in both the United States and Canada, where would a customer get the most for his money?

_____the United States, _____Canada

5. In the United States surveyors often measure length in links, chains, furlongs, and miles, where:

a <u>link</u> is a little less than 8 inches
a <u>chain</u> is 100 links
a <u>furlong</u> is 10 chains
a <u>mile</u> is 8 furlongs

With each of the following pairs, which is the greater length?

(a) _____88 links or _____1 chain

(b) _____10 inches or _____1 link

(c) _____2 miles or _____11 furlongs

(d) _____2 furlongs or _____23 chains

6. For some unknown reason, eggs and the number 12 seem to be related. Eggs are usually bought and sold by the following system:

a dozen is 12 eggs
a gross is 12 dozen
a great gross is 12 gross (12 dozen dozen)

How many eggs in a gross? ____36, ____144, ____120

How many eggs in a great gross? ____120, ____1728

Imagine that an egg dealer has the following kinds of boxes:

boxes that hold a dozen eggs
boxes that hold a gross eggs
boxes that hold a great gross eggs

When the dealer puts eggs in boxes, he keeps three things in mind:

(1) He wants to use as few boxes as possible.

(2) He never begins to fill a box unless he has enough eggs to fill it completely.

(3) He wants as few eggs left over as possible.

For each of the following numbers of eggs, try to decide how many boxes the dealer will use and how many eggs will be left over:

	Number of eggs	Number of boxes	Number of eggs left
(a)	26	____	____
(b)	200	____	____
(c)	1730	____	____
(d)	2064	____	____

7. Druggists often use the following system for weights since they work with very small weights:

a grain is very small, just a bit more than nothing
a scruple is 20 grains
a dram is 3 scruples
an ounce is 8 drams
a pound is 12 ounces

Does the druggist use the same kinds of pounds and ounces as the grocer? ____yes, ____no

Chapter Three
(Measurement using three units within a system)

In the first part of this chapter you will be asked to imagine that you are living many years ago, just as measurement of length is beginning to become a bit precise. We will imagine that we are living with a tribe which doesn't even know that there may be other tribes around. Consequently, there is, for the moment, no need to worry about having the same system as everyone else, mainly because, as far as we're concerned, there isn't anyone else. The units for measuring length are as follows:

The <u>MOUT</u> The "mout" is the length of a special mouse's tail.

The mouse whose tail was used to decide how long the "mout" should be was a very famous mouse who once bit and destroyed a poisonous scorpion just as the scorpion was about to sting a high village official during a political speech.

The <u>ELT</u> The "elt" is the length of a special elk's tail.

The elk whose tail was used to decide how long the "elt" should be was also famous. He once single-handedly (double-hornedly, if you prefer) saved the tribal food supply from disaster by driving off about a thousand starving coyotes,..... give or take a few.

The <u>STOT</u> The "stot" is short for "stone throw."

The "stot" was the distance the vice-chief's granddaughter threw a stone on her fifth birthday.

Although no thought was given to making the system easy to work with while it was being developed, the following relationships just happened to hold between the strange units:

9 MOUTS = 1 ELT

5 ELTS = 1 STOT

Just for practice work the following problems:

(a) Change 4 stots to elts.

(b) Change 3 elts to mouts.

(c) Change 30 mouts to lowest denomination using mouts and elts.

(d) Change 45 mouts to lowest denomination using mouts and elts.

(e) Change 53 elts to lowest denomination using elts and stots.

As was the case with squints and thrumps, sometimes it is desirable to put measurements in lowest denomination. There is an added problem here. We now have three units instead of two, and things are likely to become a bit more messy, not really much harder. Before we bother to define what we mean by "lowest denomination," we will work some problems that might come up using our new system. We will call the new system the "caveman system."

The only thing that is sold by length in our village is calking. Calking is used to patch cracks that develop in hut roofing, and it can be purchased in three lengths:

strips 1 mout long,

strips 1 elt long,

strips 1 stot long

Since calking dries and becomes worthless very quickly, it is never a good idea to buy any more than is needed at any time.

Naturally, no one wants to spend any more on calking than is absolutely necessary. And this is where the problems begin.

A strip 1 elt long costs a little more than 8 strips each 1 mout long, but a strip 1 elt long costs less than 9 strips each 1 mout long. In other words, if 8 mouts or less of calking is needed, it pays to buy strips a mout long. If an elt or more is needed, it does not pay to buy all strips a mout long.

The same sort of thing is true for elts and stots. A strip 1 stot long costs a little more than 4 strips each 1 elt long, but a strip 1 stot long costs less than 5 strips each 1 elt long.

For each of the following examples, decide if the customer ordered wisely if he wants to save as much money as possible:

(Write "yes" if the order is wise, "no" if more than necessary was spent or the order is for the wrong amount.)

(a) Needed: 10 mouts of calking
Ordered: 10 mout strips ________(a)

(b) Needed: 11 mouts of calking
Ordered: 1 elt strip, 2 mout strips ________(b)

(c) Needed: 6 mouts of calking
Ordered: 1 elt strip ________(c)

(d) Needed: 45 mouts of calking
Ordered: 1 stot strip ________(d)

(e) Needed: 26 mouts of calking
Ordered: 3 elt strips ________(e)

(f) Needed: 58 mouts of calking
Ordered: 1 stot strip, 1 elt strip,
4 mout strips ________(f)

(g) Needed: 90 mouts of calking
Ordered: 1 stot strip, 4 elt strips,
9 mout strips ________(g)

(h) Needed: 80 mouts of calking
Ordered: 1 stot strip, 1 elt strip,
1 mout strip ________(h)

Which rule would tell you how to order in such a way that the least will be spent?

____(a) Write the last digit as the number of mout strips, the next digit back as the number of elt strips, and the remaining digits as the number of stot strips.

____(b) Divide the number of mouts by 9. The remainder will be the number of mout strips and the quotient will give a number of elts. Then divide the number of elts by 5. The remainder will be the number of elt strips and the quotient will be a number of stots. Since there is no unit bigger than a stot, the number of stots will be the number of stot strips needed.

____(c) Subtract 9 from the number of mouts. Record the last digit of the difference as the number of mout strips needed. Record the first digit of the difference as the number of stot strips needed.

To reduce measurements to "lowest denomination" is still very much the same as it was in chapter one. In the case of three units of measure, we want as few of the small units as possible, as few of the middle-sized units as possible, and as many of the big units as possible. Rule (b) above gives the correct method for reducing the measurements to "lowest denomination."

Reduce each of the following to lowest denomination:

(a) 10 mouts = ____stots, ____elts, ____mouts

(b) 18 mouts = ____stots, ____elts, ____mouts

(c) 45 mouts = ____stots, ____elts, ____mouts

(d) 55 mouts = ____stots, ____elts, ____mouts

(e) 8 mouts = ____stots, ____elts, ____mouts

(f) 90 mouts = ____stots, ____elts, ____mouts

(g) 36 mouts = ____stots, ____elts, ____mouts

(h) 1 mout = ____stots, ____elts, ____mouts

Sometimes the measurement may be given as a number of elts. In that case there is no reason to worry about mouts. And since we want as few mouts as possible for writing the number in lowest denomination, there is certainly no reason to think about mouts. If the measurement is given as a number of stots, there is not a thing left to be done, since then we will have zero mouts and zero elts. It's pretty hard to improve on zero as a small number of mouts or elts.

Reduce each of the following to lowest denomination:

(a) 12 mouts = _____stots, _____elts, _____mouts

(b) 6 elts = _____stots, _____elts, _____mouts

(c) 63 stots = _____stots, _____elts, _____mouts

(d) 5 mouts = _____stots, _____elts, _____mouts

(e) 20 elts = _____stots, _____elts, _____mouts

(f) 990 mouts = _____stots, _____elts, _____mouts

(g) 478 stots = _____stots, _____elts, _____mouts

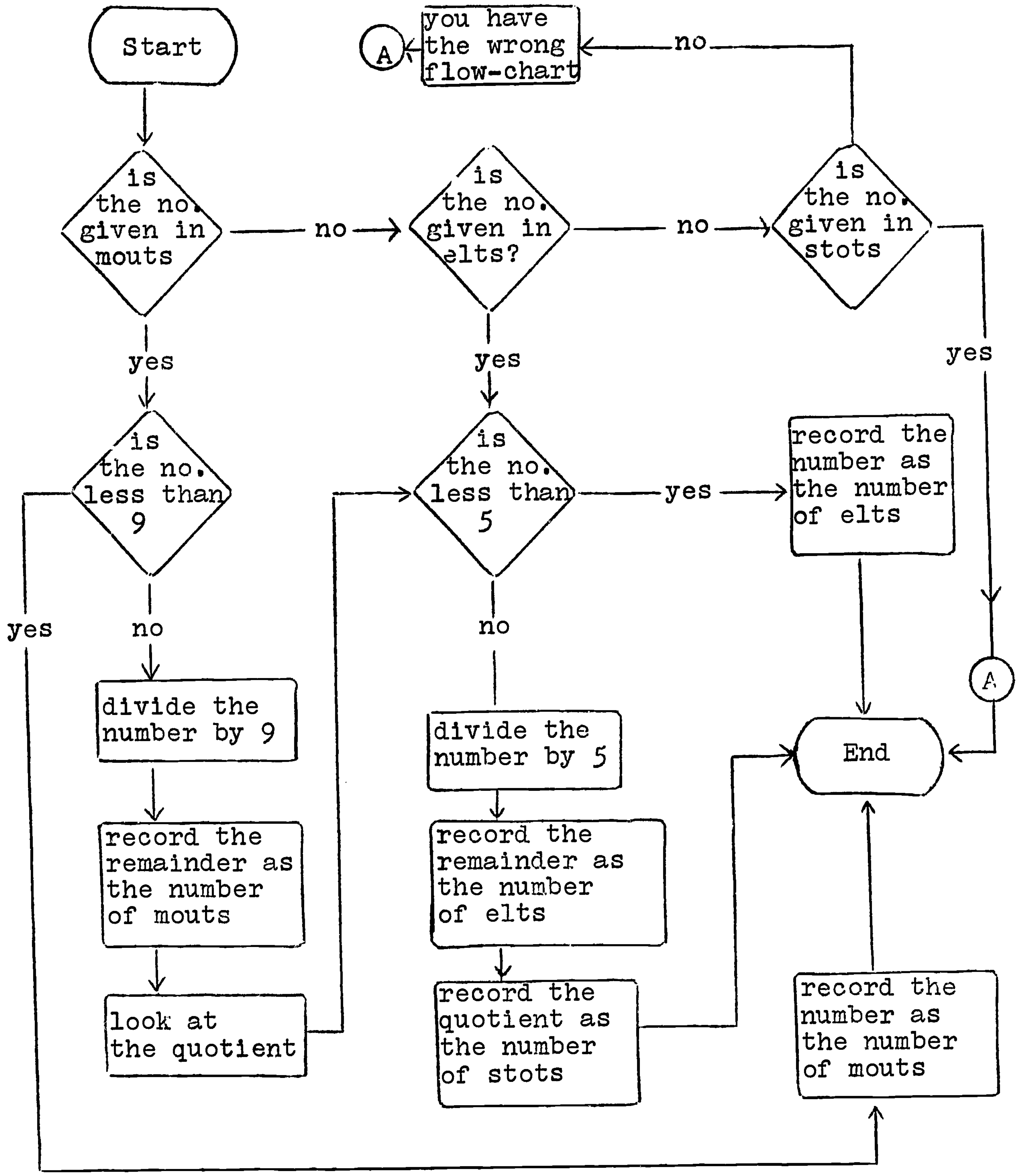
(h) 478 elts = _____stots, _____elts, _____mouts

(i) 478 mouts = _____stots, _____elts, _____mouts

Perhaps you have noticed that when a measurement is given, there is never any need to worry about units that are smaller than the units in which the measurement is given. If the measurement is given in stots, it is already in lowest denomination. If the measurement is given in elts, we only need to divide by 5 to get a number of elts and stots. If the number is given as a number

of mouts we must first change to mouts and elts, then change the elts to elts and stots.

If we were to make a flow-chart for changing measurements to lowest denomination, it might look like the following:



Using the flow-chart on page 34, change each of tb following to lowest denomination:

(a) 3 stots = _____stots, _____elts, _____mouts

(b) 8 elts = _____stots, _____elts, _____mouts

(c) 9 mouts = _____stots, _____elts, _____mouts

(d) 99 mouts = _____stots, _____elts, _____mouts

(e) 56 elts = _____stots, _____elts, _____mouts

If you look back at the flow-charts on pages 20 and 21, you will notice that these flow-charts work for any system using two units. The flow-chart we just made for the cave-man system works well for the cave-man system, but it wouldn't work for feet, inches, and yards; or, for pounds, ounces, and tons, etc. Somehow, we must make a flow-chart which works for any measurement system using three units.

Before we try to streamline our flow-chart, look at the table below:

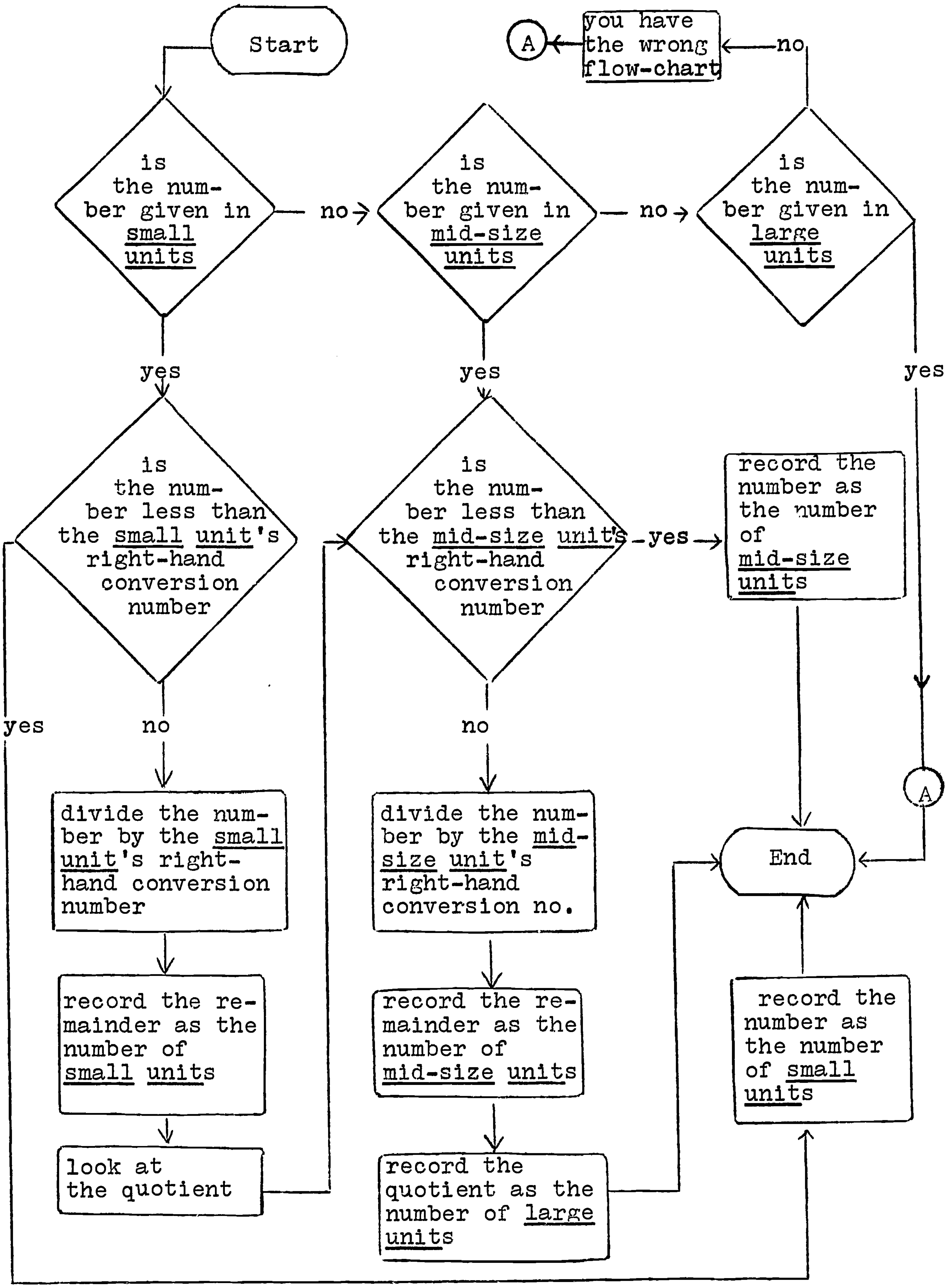
cave-man system	MOUT	9	ELT	5	STOT
English system	INCH	12	FOOT	3	YARD
angular measure	SECOND	60	MINUTE	60	DEGREE
liquid measure	PINT	2	QUART	4	GALLON
FOR ANY SYSTEM	SMALL UNIT	X	MIDDLE-SIZE UNIT	Y	LARGE(BIG) UNIT

As you can see from the table on the last page, there are two conversion numbers for each system. One of the first things we must decide is what to call the conversion numbers. (In the case of two units it was easy because there was only one conversion number; we simply called it the conversion number.)

If we agree to write measurement systems as we did on the last page, things will be easy to keep straight. We will call "12" the "<u>right-hand conversion number</u>" for inch. At the same time "12" is the "<u>left-hand conversion number</u>" for foot. "2" will be the "right-hand conversion number" for pint, and "2" will be the "left-hand conversion number" for quart. In general, looking at the last row on the chart on the last page, "Y" is the "right-hand conversion number" for MIDDLE-SIZE UNIT, and "Y" is the "left-hand conversion number" for LARGE UNIT.

In a system with three units, the small unit never has a left-hand conversion number, and the large unit never has a right-hand conversion number. (You probably won't find these definitions in any other book about measurement, but that doesn't really matter. As soon as you become very good at measurement, you can forget the definitions because all the flow-charts will become somewhat second nature to you.)

On the next page you will find a flow-chart which is identical to the one on page 34 except that we will not tell which system we're using. We will replace "mout" by "small unit," replace "elt" by "middle-size unit," and "stot" by "large unit."


Start
is the number given in small units
no
is the number given in mid-size units
no
is the number given in large units
no
you have the wrong flow-chart
A
yes
is the number less than the small unit's right-hand conversion number
yes
is the number less than the mid-size unit's right-hand conversion number
yes
record the number as the number of mid-size units
yes
yes
no
divide the number by the small unit's right-hand conversion number
record the remainder as the number of small units
look at the quotient
no
divide the number by the mid-size unit's right-hand conversion no.
record the remainder as the number of mid-size units
record the quotient as the number of large units
A
End
record the number as the number of small units

For working the following exercises you may use both the flow-chart on page 37 and the table on page 35. After working several problems, you may find that you don't need the flow-chart at all.

(a) 4 yards = _____yards, _____feet, _____inches

(b) 44 mouts = _____stots, _____elts, _____mouts

(c) 56 pints = _____gallons, _____quarts, _____pints

(d) 3 quarts = _____gallons, _____quarts, _____pints

(e) 5 quarts = _____gallons, _____quarts, _____pints

(f) 543 yards = _____yards, _____feet, _____inches

(g) 543 feet = _____yards, _____feet, _____inches

(h) 543 inches = _____yards, _____feet, _____inches

Sometimes it may be desired to have a number of larger units expressed as a number of smaller units. For example, you may want to know how many inches there are in 5 yards. We already know how to change yards to feet. (Look back at page 20.) And we also know how to change feet to inches by the same method.

Which would tell you how to change yards to inches?

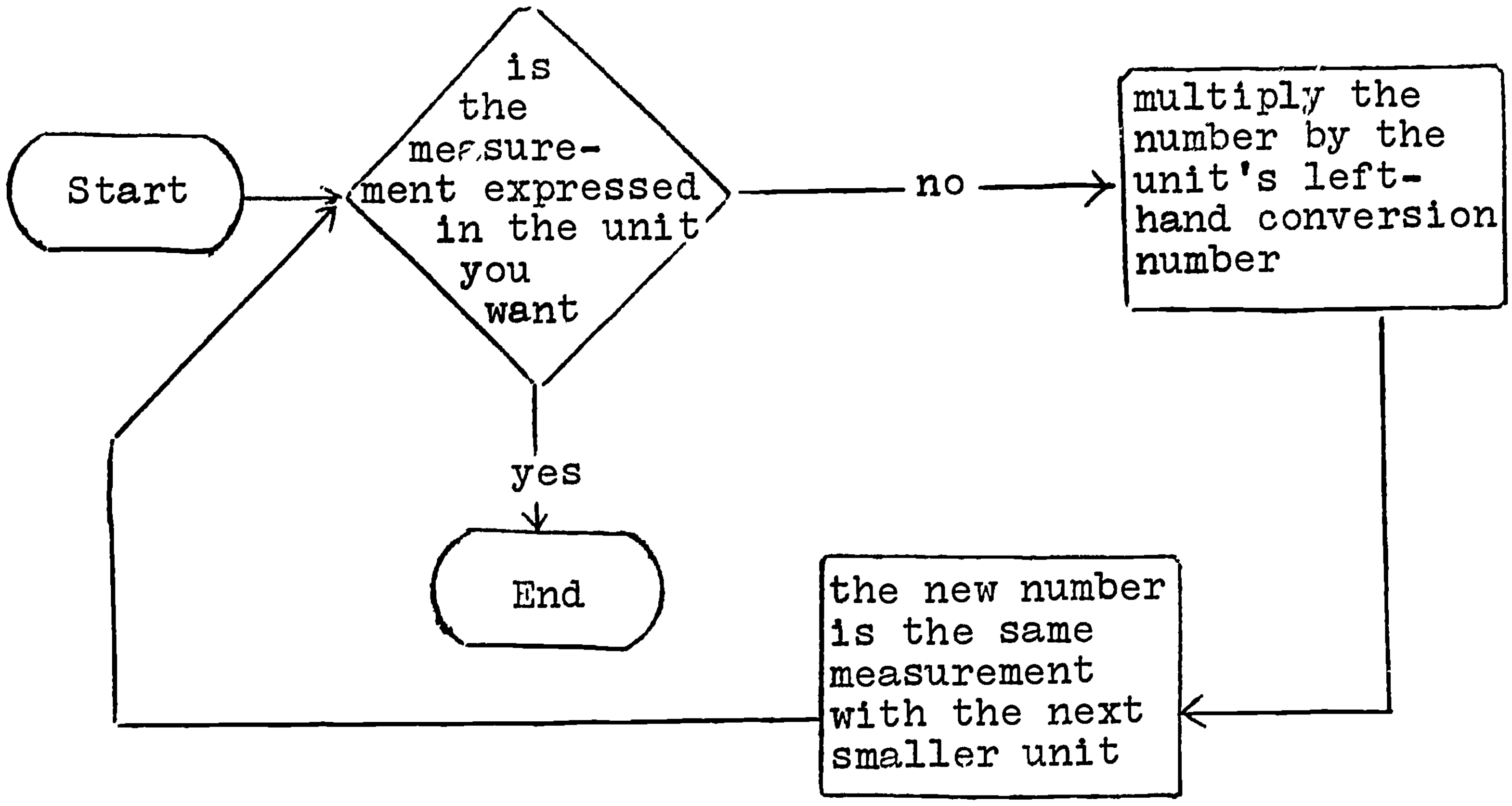
_____(a) Multiply the number of yards times 3, and then multiply the number of feet times 12.

_____(b) Multiply the number of yards times 12, and then divide by 3.

_____(c) Divide the number of yards by 3 and then divide the number of feet by 12.

_____(d) Divide the number of yards by 3. Record the remainder as the number of yards and multiply the quotient by 12.

If we wanted a general flow-chart for changing larger units to smaller units, it might look something like the following:



Using the flow-chart above and the table on page 35, change each of the following measurements:

(a) 3 yards = ________________ inches

(b) 5 gallons = ________________ quarts

(c) 5 gallons = ________________ pints

(d) 5 quarts = ________________ pints

(e) 10 stots = ________________ mouts

(f) 3 feet = ________________ inches

(g) 60 degrees = ________________ minutes

(h) 10 degrees = ________________ seconds

(i) 9 elts = ________________ stots

(j) CLASS DISCUSSION Would the flow-chart above work even if there were more than three units in the system of measurement? (For example, could it be used to change gallons to gills?)

So far we have been careful never to give more than one unit in measurements with which we worked. Things could be more difficult, and they frequently are. For example, how would we change a measurement like (2 stots, 6 elts, 14 mouts) to lowest denomination? It is clear that the measurement is not in lowest denomination as it is given since there are more than 8 mouts and there are more than 4 elts.

Circle the letter for the following which have been correctly changed to lowest denomination:

(a)	2 stots, 6 elts, 14 mouts	=	3 stots,	2 elts,	5 mouts
(b)	6 stots, 47 mouts	=	6 stots,	4 elts,	7 mouts
(c)	5 elts, 5 mouts	=	1 stots,	0 elts,	5 mouts
(d)	30 stots, 4 elts, 19 mouts	=	31 stots,	1 elts,	1 mouts
(e)	4 stots, 8 elts	=	5 stots,	7 elts,	0 mouts

Which rule would you choose as being the best method for changing messy measurements to lowest denomination?

____(a) First change each of the parts to lowest denomination. Then add the mouts together, next add the elts together and, finally, add the stots together.

____(b) Divide the number of mouts by 9, record the remainder as the number of mouts and add the quotient to the elts. Then divide the new number of elts by 5; record the remainder as the number of elts and add the quotient to the number of stots.

____(c) Divide the number of elts by 5; record the remainder as the number of elts and add the quotient to the number of stots. Next, divide the number of mouts by 9, record the remainder as the number of mouts and add the quotient to the number of elts.

NOTE: If you are not sure how to start, you may check the three methods on part (d) above. Part (d) is correct and only the best method will give the correct answer for part (d).

If none of the rules on the bottom of the last page seemed very accurate, it's probably because all of the rules talked about all three units whereas some of the examples on the middle of the last page used only two of the three units. There is an easy way to get rid of this problem; whenever only one or two of the units are given we may think of "zero" as the number with the unmentioned units. In that case we could think of 4 stots, 8 elts as being 4 stots, 8 elts, 0 mouts. Similarly, we can think of 5 elts, 5 mouts as being 0 stots, 5 elts, 5 mouts.

Using rule (b) which appeared at the bottom of the last page, change each of the following to lowest denomination:

(a) 1 stot, 8 elts, 23 mouts = ____stots, ____elts, ____mouts

(b) 465 mouts = ____stots, ____elts, ____mouts

(c) 5 stots, 45 mouts = ____stots, ____elts, ____mouts

(d) 4 stots, 50 elts, ____stots, ____elts, ____mouts

(e) 9 elts, 9 mouts = ____stots, ____elts, ____mouts

(f) 2 stots, 4 elts, 8 mouts = ____stots, ____elts, ____mouts

(g) 123 elts, ____stots, ____elts, ____mouts

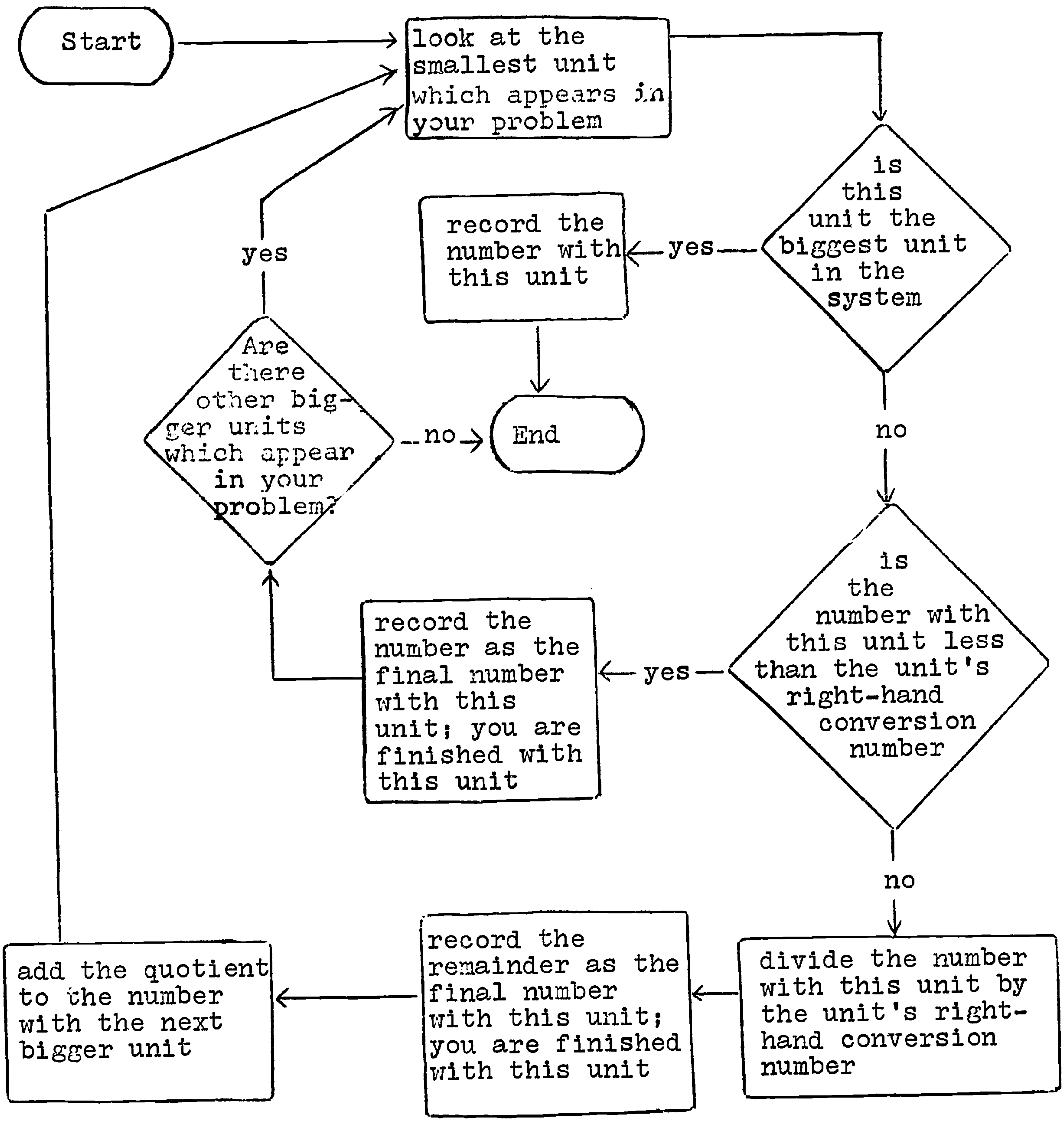
(h) 33 stots, ____stots, ____elts, ____mouts

(i) 12 stots, 34 elts, 63 mouts = ____stots, ____elts, ____mouts

(j) 1 stot, 5 elts, 10 mouts = ____stots, ____elts, ____mouts

You have very likely decided that when there are no "mouts" there is really not much reason to look at mouts. The flow-chart on the next page tries to make it possible for you to begin with the smallest unit which has a number different from zero.

A flow-chart for reducing measurements to lowest denomination:



CLASS DISCUSSION

(a) Would this flow-chart work for any system with three units?

(b) Would this flow-chart work for any system, no matter how many units there were?

After you study the flow-chart on page 42 you will probably agree that we no longer need many of the flow-charts which appeared earlier in this book. The following flow-charts are not needed if we have the one on page 42: (top of page 14, page 19, page 21, page 34, page 37)

On the less pleasant side, we suddenly realize that we do not have a suitable flow-chart for changing messy measurements to measurements expressed by a single unit. For example, how would we change 4 stots, 3 elts, 5 mouts to a measurement expressed only as a number of mouts?

Decide which of the follwoing are correct:

(a)	4 stots, 3 elts, 5 mouts	=	435 mouts
(b)	40 stots	=	1800 mouts
(c)	60 elts	=	12 mouts
(d)	10 stots, 10 elts, 10 mouts	=	550 mouts
(e)	32 stots, 4 elts	=	52 elts
(f)	2 stots, 13 elts	=	207 elts

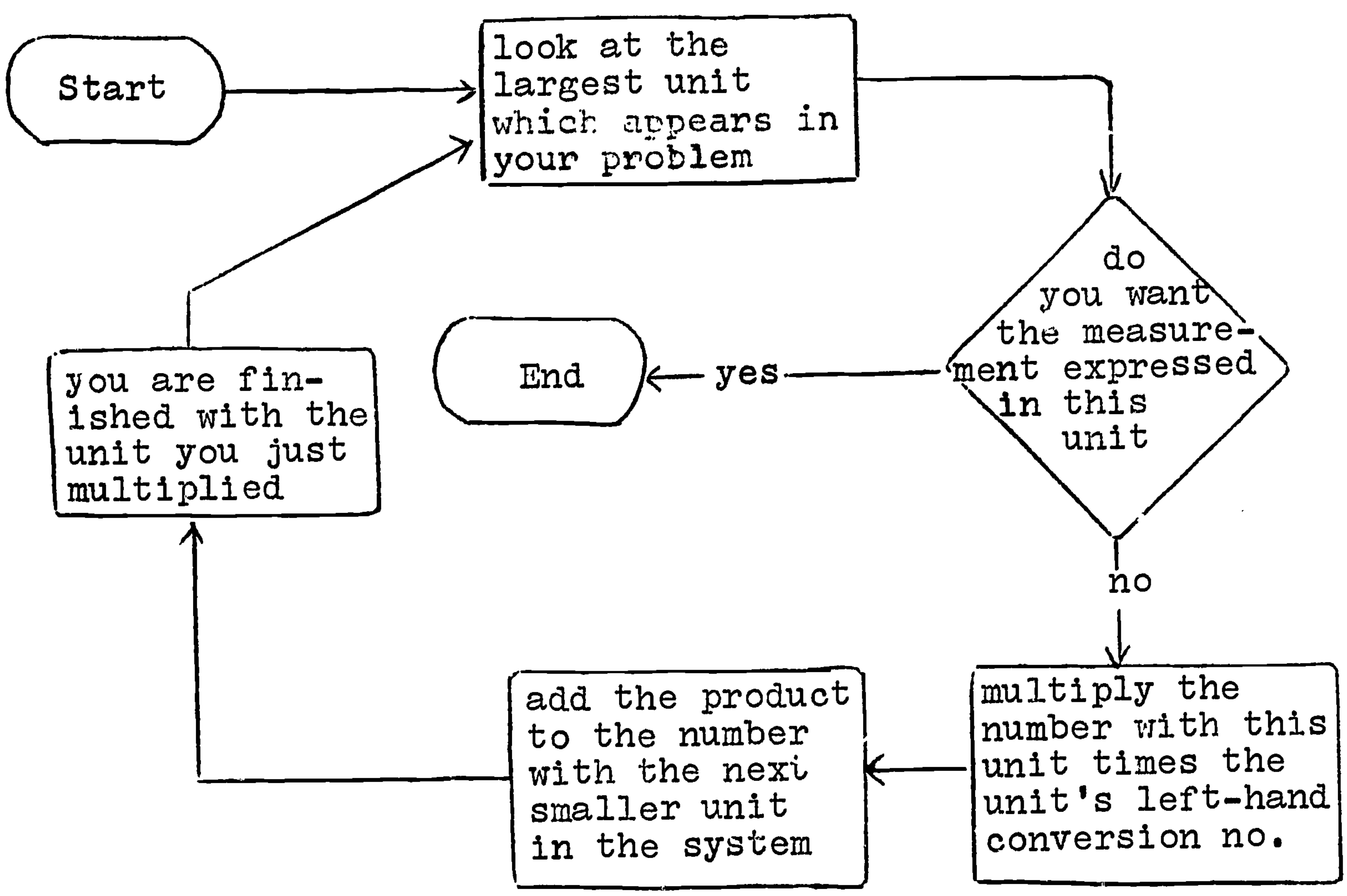
Which of the following rules seems best for the problems?

____(a) Change mouts to mouts and elts; then change elts to elts and stots; then add mouts to mouts, elts to elts, and stots to stots.

____(b) Multiply stots times 5 and add the product to the number of elts. Next multiply elts times 9 and add the product to the number of mouts.

____(c) Multiply the number of mouts times 9 and add the product to the number of elts. Next multiply the number of elts times 5 and add the product to the number of stots.

It is very unlikely that anyone would be asked to change a messy measurement to a measurement expressed by a single unit where the unit is larger than the smallest unit used in giving the messy measurement. For example, you wouldn't be asked to change 4 stots, 3 elts, 5 mouts to elts, mainly because it is impossible to express the messy measurement as a number of elts. With that in mind, the following flow-chart will perhaps be adequate for changing messy measurements to measurements expressed by a single unit:



CLASS DISCUSSION

(a) Would the flow-chart above work for any system which has three units?

(b) Would the flow-chart above work for any system, no matter how many units the system had?
(For example, could it be used to change 2 gallons, 2 quarts, 1 pint, 1 gill to a measurement expressed in gills?)

Using the flow-chart on the last page, express each of the following measurements using only the unit given to the right of the equal sign:

(a)	4 stots,	5 elts		=	________ elts
(b)	2 yards,	2 feet		=	________ feet
(c)	3 yards,	1 foot,	8 inches	=	________ inches
(d)	2 yards,		4 inches	=	________ inches
(e)	2 stots,	4 mouts		=	________ mouts
(f)		5 quarts,	3 pints	=	________ pints
(g)		4 quarts,	3 pints	=	________ gills
(h)	2 degrees,	5 minutes		=	________ minutes
(i)	4 stots,	5 elts		=	________ mouts
(j)	4 stots			=	________ mouts

CLASS DISCUSSION Decide which flow-charts in this booklet are no longer needed once we have the flow-chart on page 44.

Chapter FOUR
(Measurement using any number of units within a system)

You will soon find that there is really very little to say in this chapter. We will see that the flow-charts given on pages 42 and 44 are as good for any number of units as they were for just three units. Changing a measurement to "lowest denomination" still means expressing the measurement so that there will be as many as possible of the largest unit and as few as possible of all other units. If you look at the flow-chart on page 42 you will see that there is nothing said about the number of units in the system. Also, the only ways to stop going around and around in the flow-chart are to either get all the way to the biggest unit in the system or to work until there aren't any bigger units left to work with.

The same things are true for the flow-chart on page 44. Again, there is no mention made of the number of units in the system. The only way to stop going around and around is to get to the unit you want for expressing the measurement, no matter how many steps it takes.

The most important thing in working with many units in a system is patience. The problems in this section won't be any different from those in the last chapter, only longer. There won't be any need for new flow-charts for the problems. (If you do use the flow-charts, try not to become dizzy from going around so often.)

The following information may be used for working problems. As you already know, there are many systems of measurement in the world, and almost no one remembers all the names of the units or the conversion numbers. (Only a few of the systems appear on this page.) As before, the units will be given from smallest to largest and the numbers between the units will tell how many of the smaller units there are in one of the larger ones.

(ENGLISH) LINEAR MEASURE
inch, 12, foot, 3, yard, 1760, mile

(ENGLISH) SQUARE MEASURE
square inch, 144, square foot, 9, square yard

LAND MEASURE
centiare, 100, are, 100, acre, 100, hectare

TROY MEASURE
grain, 24, pennyweight, 20, ounce, 12, pound

APOTHECARIES' WEIGHT
grain, 20, scruple, 3, dram, 8, ounce, 12, pound

(COMMON) WEIGHT MEASURE
ounce, 16, pound, 200, ton

LIQUID MEASURE
gill, 4, pint, 2, quart, 4, gallon

CHAIN (SURVEYOR'S) MEASURE
link, 100, chain, 10, furlong, 80, mile

(METRIC) LINEAR MEASURE
millimeter, 10, centimeter, 10, decimeter, 10, meter, 10, decameter, 10, hectometer, 10, kilometer

(ENGLISH) CUBIC MEASURE
cubic inch, 1728, cubic foot, 27, cubic yard

DRY MEASURE
pint, 2, quart, 8, peck, 4, bushel

NAUTICAL MEASURE
foot, 6, fathom, 100, cable length, 10, nautical mile

You may use the flow-chart on page 44 and the information on page 47 to work the following:

(a) 2 yards = __________inches

(b) 1 mile = __________feet

(c) 20 hectares = __________acres

(d) 5 pennyweights = __________grains

(e) 2 bushels = __________pints

(f) 1 kilometer = __________millimeters

(g) 1 yard, 2 feet = __________feet

(h) 2 yards, 1 foot = __________inches

(i) 5 furlongs, 1 chain, 3 links = __________links

(j) 1 pound, 1 ounce, 1 dram = __________scruples

(k) 2 gallons, 2 quarts, 2 pints, 2 gills = __________gills

(l) 1 nautical mile = __________feet

(m) 3 furlongs, 2 chains = __________links

(n) 1 square yard = __________square inches

(o) 1 cubic yard = __________ square inches

CLASS DISCUSSION QUESTIONS

(a) Do you think that there are more than enough systems of measurement for the uses we have for measurement?

(b) Which is more important, to become good at one or two systems or to become good at knowing the properties which all systems have in common?

(c) Look at page 47 and comment on which systems of measurement seem to be well thought out and on which systems of measurement look as though they came about without much thought.

You may use the flow-chart on page 42 and the information on page 47 to work the following. Change each measurement to lowest denomination.

(a) 144 inches =

(b) 144 square inches =

(c) 327 acres =

(d) 19 pecks =

(e) 3546 fathoms =

(f) 5 feet, 15 inches =

(g) 17 pints, 38 gills =

(h) 21 grains, 12 scruples, 1 dram =

(i) 600 links, 6 furlongs =

(j) 1730 cubic inches, 1 cubic foot =

(k) 6000 feet, 1000 fathoms =

(l) 1000000 millimeters =

(m) 1253694 millimeters =

(n) 486 meters, 58 decameters, 2 hectometers =

(o) 1000000 centiares =

By this time you must be very weary of measurement. The sad fact is that we need measurement for so many things that we do.

CLASS DISCUSSION As a class discuss what could be done to either change systems we have, get rid of systems which no one really needs, and replace systems by neater systems which would have units close enough to the desired sizes so that no serious problems would arise. (For example, you couldn't very well ask the earth-moving business to adopt the Apothecaries' Weight System since all of the units would be much too small for easy use.)

CLASS DISCUSSION Why do you think scientists usually use the metric system for length, weight, and volume?

CLASS DISCUSSION If England uses only the Metric System and the United States uses only the English System, what problems come up when the two countries want to buy and sell between each other?

CLASS DISCUSSION How difficult would it be for this country to change to the metric system, say only for length? (Think particularly of things which would have to be changed, how difficult it would be to think in a new system, and how many things which now work out nicely would then have fractions or decimals where we don't want them.)

Chapter FIVE
(Addition and subtraction using measurements)

Before we state any rules for adding or subtracting measurements, decide which of the following statements are true and which are false:

CLASS DISCUSSION

(a) 3 Buicks + 5 Lincolns + 2 Hondas = 10 cars

(b) 12 inches + 5 centimeters = 17 decimeters

(c) 5 feet + 3 feet + 2 feet = 10 feet

(d) 6 rabbits + 2 Beagles = 2 Beagles

(e) 3 inches + 6 inches + 3 inches = 1 foot

You most likely agree that some of the statements above are sheer nonsense. (a) is clearly false since a Honda is not a car. To see that (b) is false one only has to take a ruler and a meter stick to see that it isn't too accurate to add inches to centimeters and then call the result decimeters. On the other hand, a ruler will make it plain that (c) is true. (d) presents problems; people who know that Beagles are excellent rabbit hounds would very likely say that (d) is true, but mathematically the statement makes very little sense. (e) is true only because we already know that 12 inches is the same as 1 foot.

In general, we can add quantities only if all the quantities are expressed by the same unit of measure. It makes good sense to add pounds and pounds and then call the sum a number of pounds. But it makes no sense to add pounds to ounces and then call the result drams or scruples. The rule is to <u>ADD ONLY QUANTITIES WHICH ARE GIVEN IN THE SAME UNIT</u>.

Before we discuss addition and subtraction of measurements we will look at ordinary addition and subtraction of numbers.

```
 75389
  5019
    46
  4982
    67
+    4
------
```

There will be nothing very shocking here, simply a look at why we do what we do when we add numbers. Looking at the addition problem to the right, we would first add the "units" column, giving us 37. To find out how many tens there are in 37 we divide 37 by 10, giving 3 with remainder 7. (This is done mentally.) We record the 7 as the number of units

```
     3
 75389
  5019
    46
  4982
    67
+    4
------
     7
```

and put the 3 with the tens. We may think of the tens as the next biggest unit for measuring "how many." The next step would be to find out how many tens there were in all. After finding out that there were 30 tens we would reduce 30 tens to sort of a lowest denomination by noticing that 30 tens is the same as 3 hundreds and 0 tens. We continue this process until nothing remains to the left.

```
  133
 75389
  5019
    46
  4982
    67
+    4
------
 85507
```

Addition of messy measurements can be done in very much the same way. As with numbers, we begin with the smaller units since many smaller units will make bigger units if we want lowest denomination; but, if we want lowest denomination, we never need to worry about changing larger units to smaller ones. Another thing to notice is that with numbers we seldom write "units" with the units or "tens" with tens. Everyone just assumes

the right column is for units, the next column to the left is for tens, the next column to the left is for hundreds, and so on.

To add 5 feet, 2 inches; 2 yards, 2 feet; 7 yards, 1 foot, 9 inches; and 4 feet, 3 inches, we would set up the problem as you see below:

	5 feet,	2 inches
2 yards,	2 feet	
7 yards,	1 foot,	9 inches
+	4 feet,	3 inches

As we do with ordinary addition, we begin working with the column to the right, the column with the smallest sized units. Adding, we find that there are 14 inches. Changing to lowest denomination using feet and inches this gives us 1 foot and 2 inches. We record the 2 inches and put the foot in good company with the feet:

	1 5 feet,	2 inches
2 yards,	2 feet	
7 yards,	1 foot,	9 inches
+	4 feet,	3 inches
		2 inches

Next we add the feet, just as we added tens after we had added units. Adding feet gives us 13 feet. Changing to lowest denomination using feet and yards gives us 4 yards and 1 foot. We will record the "1" below the feet and put the "4" with the yards where it belongs. As a final step we will add the yards to see that there are 13 yards. (The thing that makes this harder than ordinary addition is that in ordinary addition we always divided

by 10.) In this example we had to divide inches by 12 and we divided feet by 3.

$$\begin{array}{rrr}
 & \overset{1}{5}\text{ feet,} & 2\text{ inches} \\
\overset{4}{2}\text{ yards,} & 2\text{ feet} & \\
7\text{ yards,} & 1\text{ foot,} & 9\text{ inches} \\
+ & 4\text{ feet,} & 3\text{ inches} \\
\hline
13\text{ yards,} & 1\text{ foot,} & 2\text{ inches}
\end{array}$$

Before looking at subtraction using messy measurements, we will look at ordinary subtraction. As long as the top number in each column is at least as big as the bottom number, subtraction is easy since there is no need to borrow. The difficulty comes when the top number in one of the columns is bigger than the bottom number in that column.

In that case we do what almost anyone in need would do, we borrow. We borrow from the column to the left if there is enough there to borrow; otherwise we go further to the left until we find a place from which we can borrow.

$$\begin{array}{r}
15732 \\
-4218 \\
\hline
\end{array}
\qquad
\begin{array}{r}
157\overset{2}{\cancel{3}}\overset{12}{\cancel{2}} \\
-4218 \\
\hline
11514
\end{array}$$

$$\begin{array}{r}
1404 \\
-178 \\
\hline
\end{array}
\qquad
\begin{array}{r}
1\overset{3}{\cancel{4}}\overset{9}{\cancel{0}}\overset{14}{\cancel{4}} \\
-178 \\
\hline
1226
\end{array}$$

Almost the same things are true for subtracting messy measurements. If the top number for each measurement is at least as big as the bottom number, there will be no need to borrow, and subtraction will be easy. If the top number in a column is smaller than the bottom number, we will have to make things nicer by borrowing, and we will always go to the left to borrow.

The example to the right is one in which no borrowing is necessary.

	5 yards,	2 feet,	11 inches
−	1 yard,	2 feet,	6 inches
	4 yards,		5 inches

The next example is one in which borrowing is needed, but in which we don't have to go very far to borrow.

	7 gallons,	$\overset{2}{\cancel{3}}$ quarts,	$\overset{2}{\cancel{2}}$ pints
−	7 gallons,	1 quart,	1 pint
		1 quart,	1 pint

The last example is one in which borrowing becomes slightly complicated because we had to go quite far to the left to find a place from which to borrow.

	$\overset{7}{\cancel{8}}$ yards,	$\overset{2}{\cancel{0}}$ feet,	$\overset{15}{\cancel{3}}$ inches
−	5 yards,	2 feet,	11 inches
	2 yards,		4 inches

In general it is best to change all measurements to lowest denomination before subtracting, primarily because borrowing is less difficult when everything is in lowest denomination. Also, if no more than necessary is borrowed in each case, the answer will always be in lowest denomination.

With addition we can choose to first add and then change the answer to lowest denomination. On page 53 we changed measurements to lowest denomination as we went along. If we had wanted to, we could have first added each different unit and then changed the

answer to lowest denomination later. Sometimes, when the measurements are not given in lowest denomination, this may seem easier.

		5 feet,	2 inches
	2 yards,	2 feet	
	7 yards,	1 foot,	9 inches
+		4 feet,	3 inches
	9 yards,	12 feet,	14 inches
	(changing to lowest denomination)		
	13 yards,	1 foot,	2 inches

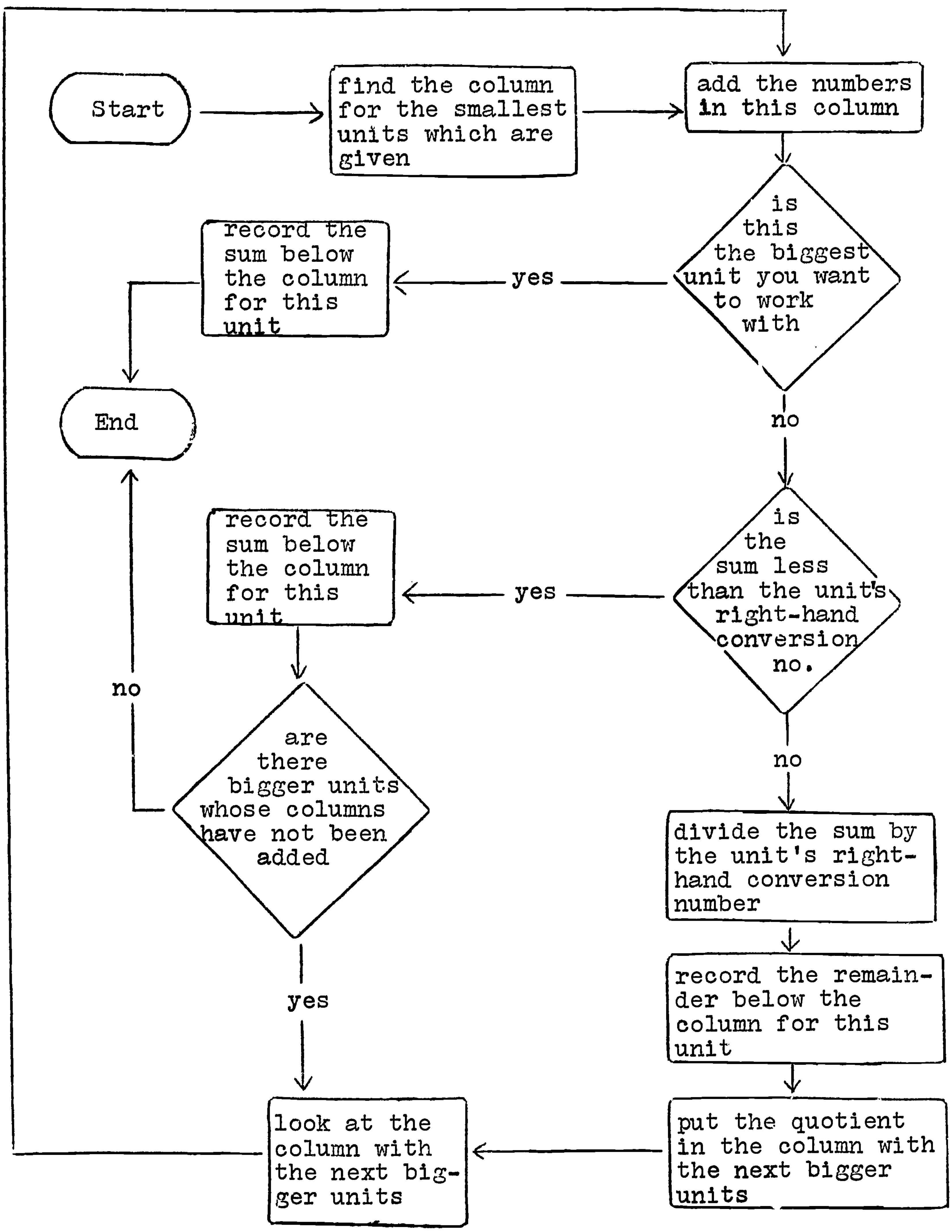
The following addition problem gives the measurements in a form which is not in lowest denomination.

	2 yards,	18 feet,	4 inches
	5 yards,	1 foot,	68 inches
	4 yards,		20 inches
		43 feet	
			15 inches
+	34 yards		
	45 yards,	62 feet,	107 inches
	(changing to lowest denomination)		
	68 yards,	1 foot,	11 inches

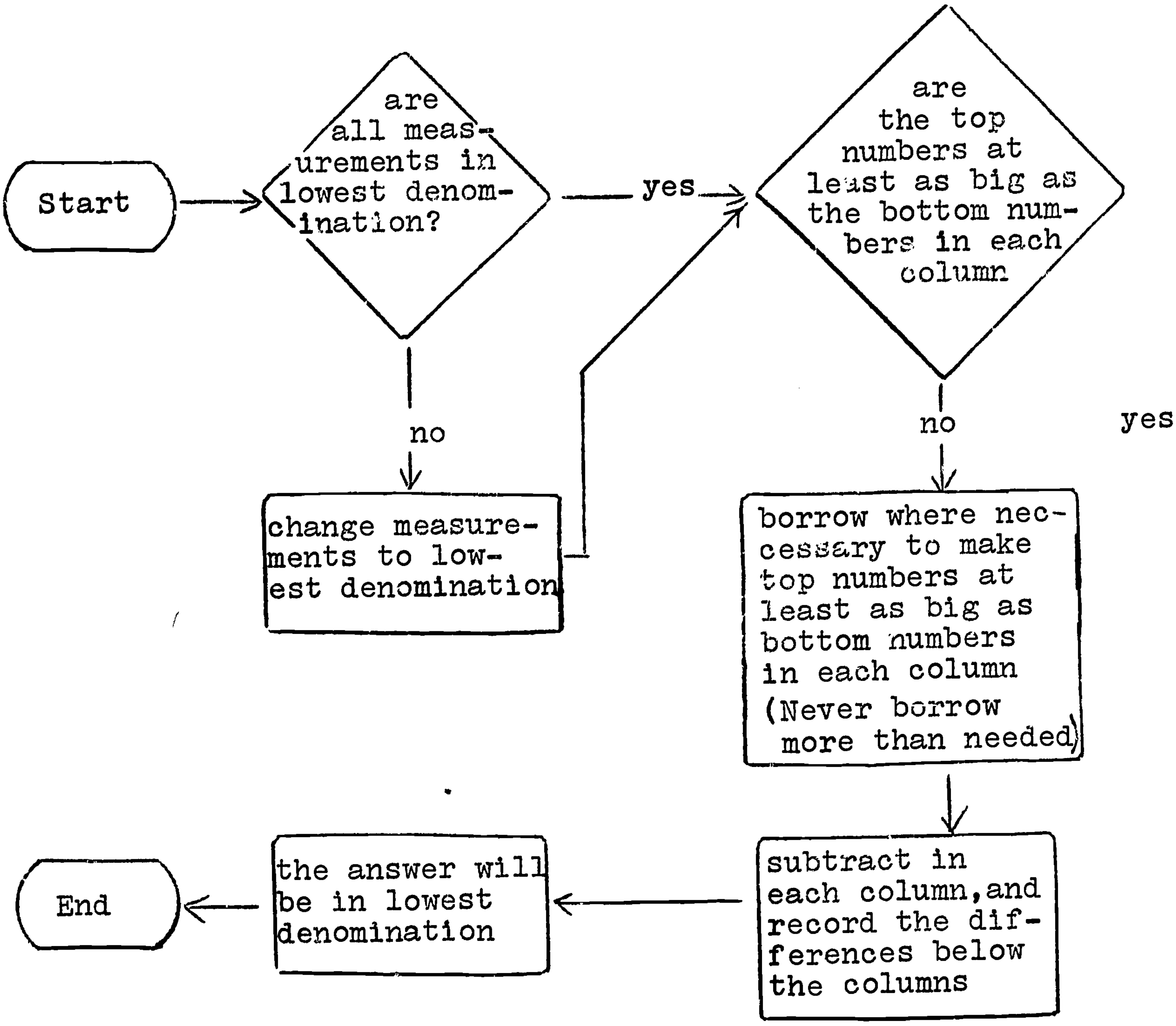
It really makes no difference which of the two methods you choose to use; both give answers equally accurate. It all depends on whether or not you like to think about adding and changing to lowest denomination at the same time, or if you prefer to first add and then think about changing to lowest denomination.

On the next pages we will try to give flow-charts for adding and subtracting messy measurements. As we have been doing, we will not mention the system or the number of units in the system. That way the flow-charts should work for any system.

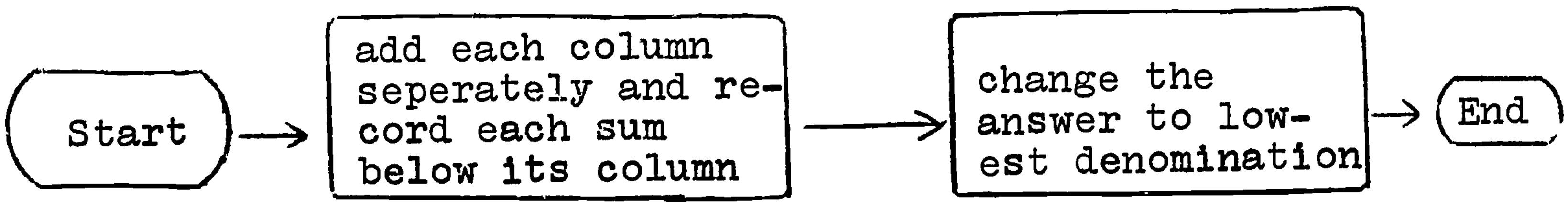
FLOW-CHART FOR ADDING MEASUREMENTS (Changing to lowest denomination as you add)



FLOW-CHART FOR SUBTRACTING MEASUREMENTS



FLOW-CHART FOR ADDING MEASUREMENTS (Changing to lowest denomination after adding)



Perform the indicated operations:

(1) 2 yards, 2 feet
\+ 7 feet

(2) 3 feet, 5 inches
\- 2 feet, 3 inches

(3) 7 links, 90 chains
67 chains
5 links
\+ 2 links, 9 chains

(4) 4 fathoms, 5 feet
1 fathom, 2 feet
23 fathoms
4 fathoms, 4 feet
\+ 25 feet

(5) 6 scruples, 18 grains
\- 19 grains

(6) 7 yards, 0 feet, 11 inches
\- 2 yards, 2 feet, 2 inches

(7) 134 tons, 5 pounds
1980 pounds
20 tons
3 tons, 540 pounds
4 pounds
\+ 1 ton, 1 pound

(8) 2 quarts, 0 pints, 2 gills
\- 1 quart, 1 pint, 3 gills

(9) 4 bushels
\- 1 bushel, 0 pecks, 3 quarts

(10) 1 mile
98 furlongs
\+ 200 chains

(11) 7 square yards, 8 square feet, 109 square inches
35 square feet
2 square yards, 3 square inches
\+ 5 square yards, 7 square feet, 31 square inches

(12) 56 cubic yards, 0 cubic feet, 90 cubic inches
\- 9 cubic yards, 14 cubic feet, 92 cubic inches

(13) 1 pound, 11 ounces, 7 drams, 2 scruples, 19 grains
\+ 1 grain

(14) 6 yards + 45 inches + 68 feet - 2 yards + 79 inches - 1 foot + 37 inches - 2 feet - 2 yards + 1890 inches = ?

CLASS DISCUSSION QUESTIONS

(1) Discuss the merits of the two methods for addition of measurements, changing to lowest denomination as you add, and first adding and then changing to lowest denomination.

(2) Discuss approaches which could be used to work problem (14) on the last page.

(3) Look at the following problem:

6 yards,	189 feet,	76 inches
- 2 yards,	180 feet,	2 inches

Is it easier to subtract first and then change the result to lowest denomination, or is it easier to first change the measurements to lowest denomination and then subtract? (Maybe both methods are equally difficult.)

Chapter SIX
(Multiplication and Division using measurements)

People disagree widely on which is the best method for multiplying and dividing measurements. Multiplication causes fewer problems than division since the numbers with each unit usually become larger and because it works very well to simply think of multiplication as repeated addition.

Thinking of multiplication as repeated addition, we have the same choice as we had with addition in the last chapter. We can first multiply (add many times) and then put the answer in lowest denomination, or we can put the product (a kind of sum) in lowest denomination as we go along.

Looking at the first method we would first multiply in each column and record the products below before worrying about lowest denomination.

3 yards,	4 feet,	7 inches
	×	8
24 yards,	32 feet,	56 inches
(in lowest denomination)		
36 yards,	0 feet,	8 inches

Using the "put it in lowest denomination as you go" method, we would first find out how many inches there were, divide by 12, record the remainder as the number of inches, and put the quotient with the number of feet.

3 yards,	4 feet,	7 inches
	×	8
		8 inches

$$\begin{array}{r} 4 \\ 12\overline{)56} \\ \underline{48} \\ 8 \end{array}$$

Next, we would multiply 8 times 4 feet to get 32 feet; to this we would add the additional 4 feet from the inches, giving 36 feet. Dividing by 3 gives us 12 yards with no feet.

$$\begin{array}{rrr} {}^{12} & {}^{4} & \\ 3 \text{ yards,} & 4 \text{ feet,} & 7 \text{ inches} \\ & \times & 8 \\ \hline & 0 \text{ feet,} & 8 \text{ inches} \end{array}$$

$$\begin{array}{r} 12 \\ 3\overline{)36} \\ \underline{36} \\ 0 \end{array} \qquad \begin{array}{r} 4 \\ 12\overline{)56} \\ \underline{48} \\ 8 \end{array}$$

The last step is to find the number of yards. Multiplying 8 times 3 yards gives 24 yards; and adding the carried 12 yards gives 36 yards.

$$\begin{array}{rrr} 3 \text{ yards,} & 4 \text{ feet,} & 7 \text{ inches} \\ & \times & 8 \\ \hline 36 \text{ yards,} & 0 \text{ feet,} & 8 \text{ inches} \end{array}$$

Division is the most difficult operation. Here we must begin work with the largest unit given and work toward the smallest. If this surprises you at first, remember that in ordinary long division with numbers we do the same thing. With addition, subtraction, and multiplication we begin work from the right (the units place) and with long division we begin at the left digit.

Once again, we can give at least two different approaches to dividing a measurement. An approach which is easier to understand would be to first change the measurement, expressing it in the smallest unit used in giving the measurement. (See the flow-chart on page 44 if your memory is on strike.) For example, if we were asked to divide 3 yards, 4 feet, 7 inches by 2 we would first change the measurement to 163 inches. We could

then divide by 2 giving us $81\frac{1}{2}$ inches. Changing 81 inches to lowest denomination gives us 2 yards, 0 feet, 9 inches. Whoops! We mustn't forget about that silly little $\frac{1}{2}$ inch. The answer is actually 2 yards, 0 feet, $9\frac{1}{2}$ inches.

The second method works very much like ordinary long division. There is one thing we must remember if we use the "long division method," we should have the measurement in lowest denomination before we begin. Putting the measurement in lowest denomination, 3 yards, 4 feet, 7 inches becomes 4 yards, 1 foot, 7 inches.

As with ordinary long division, we begin at the right of the measurement. Dividing 4 by 2 gives us 2 with no remainder.

```
    2
2/  4 yards,   1 foot,   7 inches
    4
    -
    0
```

Dividing 1 by 2 gives us 0 with remainder 1.

```
    2          0
2/  4 yards,   1 foot,   7 inches
    4          0
    -          -
    0          1
```

The remainder must now be changed to inches and added to the 7 inches we already have.

```
    4          0
2/  4 yards,   1 foot,   7 inches
    4          0        12
    -          -        --
    0          1        19
```

Finally, dividing the 19 by 2 gives us 9 with a remainder 1. Since the divisor is 2, a remainder of 1 means an additional $\frac{1}{2}$.

```
    4          0         9½
2/  4 yards,   1 foot,   7 inches
    4          0        12
    -          -        --
    0          1        19
                        18   = 2x9
                        --
                         1   = remain-
                                 der
```

The answer is 2 yards, 0 feet, $9\frac{1}{2}$ inches

CLASS DISCUSSION Examine the following example and explain what happened in each step. (When you work problems using this method, only the last line would appear. This example only shows the work step by step.)

$$\begin{array}{r|lllllll} \cline{2-8} 5 & 27 \text{ bushels,} & 1 & \text{peck,} & 6 & \text{quarts,} & 1 & \text{pint} \end{array}$$

line (1)

$$\begin{array}{r|rlrlrlrl}
 & 5 & & & & & & & \\ \cline{2-9}
5 & 27 & \text{bushels,} & 1 & \text{peck,} & 6 & \text{quarts,} & 1 & \text{pint} \\
 & \underline{-25} & & & & & & & \\
 & 2 & & & & & & &
\end{array}$$

line (2)

$$\begin{array}{r|rlrlrlrl}
 & 5 & & & & & & & \\ \cline{2-9}
5 & 27 & \text{bushels,} & 1 & \text{peck,} & 6 & \text{quarts,} & 1 & \text{pint} \\
 & \underline{-25} & & \underline{+8} & & & & & \\
 & \cancel{2} & & 9 & & & & &
\end{array}$$

line (3)

$$\begin{array}{r|rlrlrlrl}
 & 5 & & 1 & & & & & \\ \cline{2-9}
5 & 27 & \text{bushels,} & 1 & \text{peck,} & 6 & \text{quarts,} & 1 & \text{pint} \\
 & \underline{-25} & & \underline{+8} & & & & & \\
 & \cancel{2} & & 9 & & & & & \\
 & & & \underline{-5} & & & & & \\
 & & & 4 & & & & &
\end{array}$$

line (4)

$$\begin{array}{r|rlrlrlrl}
 & 5 & & 1 & & & & & \\ \cline{2-9}
5 & 27 & \text{bushels,} & 1 & \text{peck,} & 6 & \text{quarts,} & 1 & \text{pint} \\
 & \underline{-25} & & \underline{+8} & & \underline{+32} & & & \\
 & \cancel{2} & & 9 & & 38 & & & \\
 & & & \underline{-5} & & & & & \\
 & & & \cancel{4} & & & & &
\end{array}$$

line (5)

$$\begin{array}{r|rlrlrlrl}
 & 5 & & 1 & & 7 & & & \\ \cline{2-9}
5 & 27 & \text{bushels,} & 1 & \text{peck,} & 6 & \text{quarts,} & 1 & \text{pint} \\
 & \underline{-25} & & \underline{+8} & & \underline{+32} & & & \\
 & \cancel{2} & & 9 & & 38 & & & \\
 & & & \underline{-5} & & \underline{-35} & & & \\
 & & & \cancel{4} & & 3 & & &
\end{array}$$

line (6)

$$\begin{array}{r|rlrlrlrl}
 & 5 & & 1 & & 7 & & 1 & \\ \cline{2-9}
5 & 27 & \text{bushels,} & 1 & \text{peck,} & 6 & \text{quarts,} & 1 & \text{pint} \\
 & \underline{-25} & & \underline{+8} & & \underline{+32} & & \underline{+6} & \\
 & \cancel{2} & & 9 & & 38 & & 7 & \\
 & & & \underline{-5} & & \underline{-35} & & \underline{-5} & \\
 & & & \cancel{4} & & \cancel{3} & & 2 &
\end{array}$$

line (7)

$$\begin{array}{r|rlrlrlrl}
 & 5 & & 1 & & 7 & & 1\ 2/5 & \\ \cline{2-9}
5 & 27 & \text{bushels,} & 1 & \text{peck,} & 6 & \text{quarts,} & 1 & \text{pint} \\
 & \underline{-25} & & \underline{+8} & & \underline{+32} & & \underline{+6} & \\
 & \cancel{2} & & 9 & & 38 & & 7 & \\
 & & & \underline{-5} & & \underline{-35} & & \underline{-5} & \\
 & & & \cancel{4} & & \cancel{3} & & \cancel{2} &
\end{array}$$

We will now look carefully at what went on in the example given on the last page. Dividing by 5 is the same as finding out how much 1/5 of the measurement would be.

In line (1) we see that $5\overline{)27}$ is 5 with a remainder 2. In other words, 1/5 of 27 bushels is 5 and 2/5 bushels. But we don't want to have fractions of bushels. If there have to be fractions, we would like to have them only with the smallest unit. We write 5 in the quotient as the number of bushels. 5 is 1/5 of 25 bushels. We haven't yet taken 1/5 of the remaining 2 bushels. Since we haven't worked with pecks yet, we might as well take the left-over 2 bushels and put them with the pecks, and that is exactly what we do in line (2). So far we have taken 1/5 of 25 bushels.

Next, we will take 1/5 of 9 pecks where the 9 pecks came from the one peck given added to the 2 left-over bushels, which is really the same as 8 pecks. In line (3) we divide 9 by 5 to get 1 with remainder 4. This means that 1/5 of 9 pecks is 1 and 4/5 pecks. Once again, we don't want fractions of pecks. We write the 1 in the quotient. This means that we have taken 1/5 of 5 pecks. The 4 left-over pecks will be tossed in with the quarts, since we haven't yet worked with quarts. In line (4) we re-write 4 pecks as 32 quarts. Adding the 32 quarts to the 6 quarts given leaves us with a total of 38 quarts.

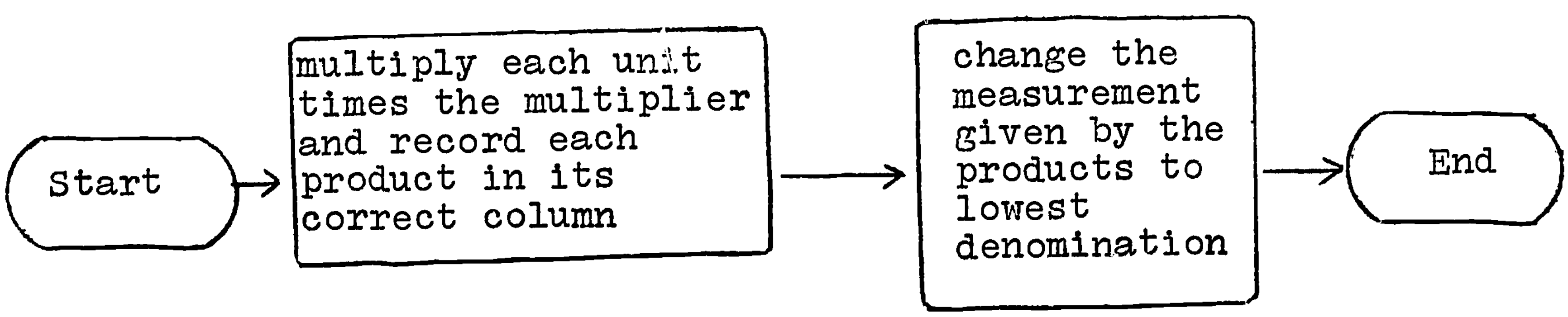
In line (5) we do much the same thing all over again. We find that 1/5 of 38 quarts is 7 with a remainder 3. Since we don't want a fraction with quarts, we settle for 7 in the quotient. This means that we haven't yet taken 1/5 of the left-over 3 quarts.

Instead of doing any more with quarts, in line (6) we change the 3 left-over quarts to pints. Adding these 6 pints to the 1 pint which was given leaves us with 7 pints. Dividing 7 pints by 5 gives us 1 pint with remainder 2.

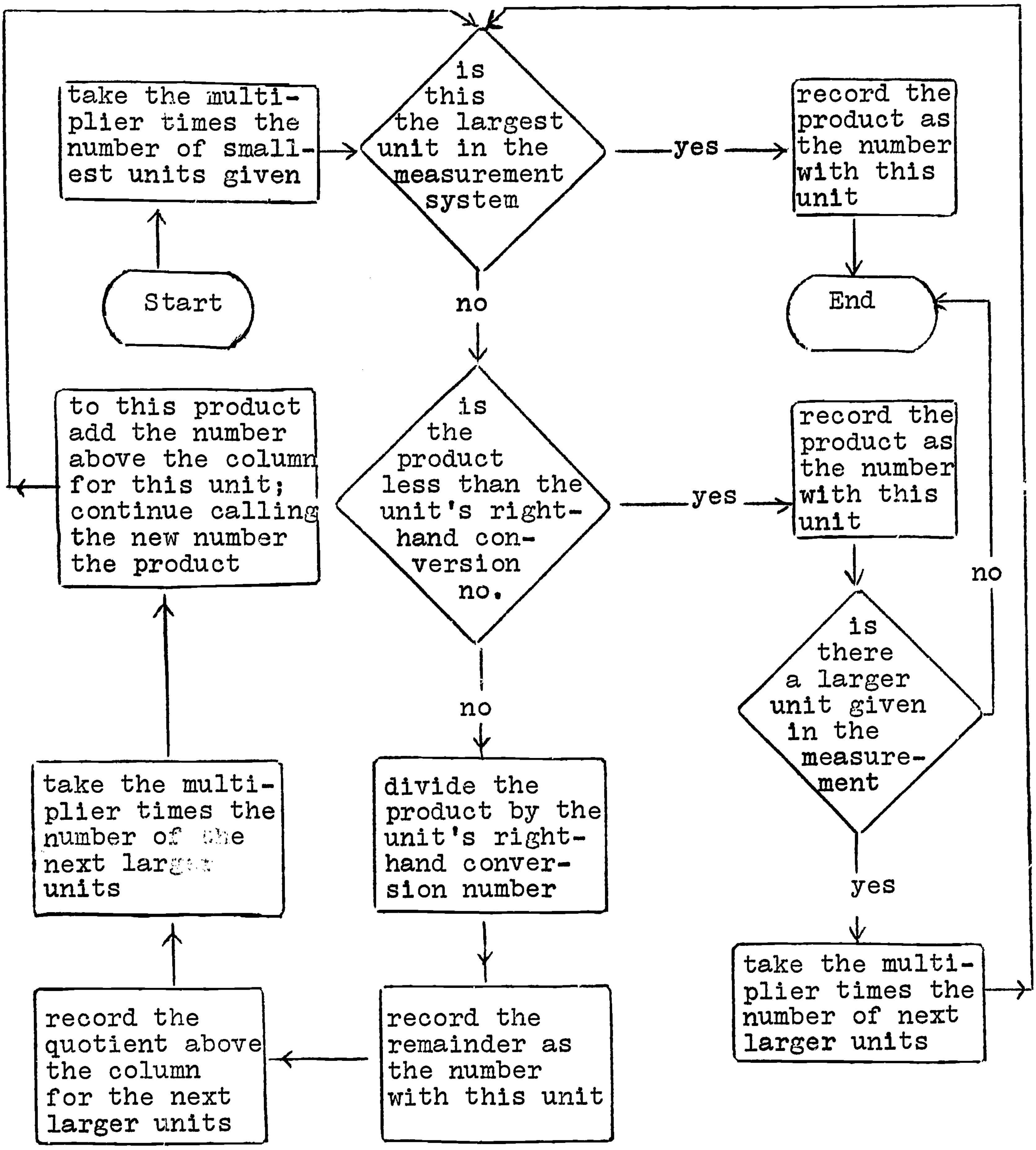
But this time we have no place to go with our remainder. We have worked with all the units, and there is no smaller unit to which we can give the 2 left-over pints. A fifth of the 2 left-over pints is a fraction, 2/5. There is no way to avoid having a fraction. Instead of fretting, we face the fact that there will be a fraction in the answer. In line (7) we put the 2/5 along with the other pint in the quotient. At least we avoided having fractions anywhere else.

As with the other operations involving measurements, we can once again make flow-charts for what we just did in which no reference to a system is made.

FLOW-CHART FOR MULTIPLYING MEASUREMENTS (Multiplying first and then changing to lowest denomination.)



FLOW-CHART FOR MULTIPLYING MEASUREMENTS (Changing to lowest denomination as you multiply)

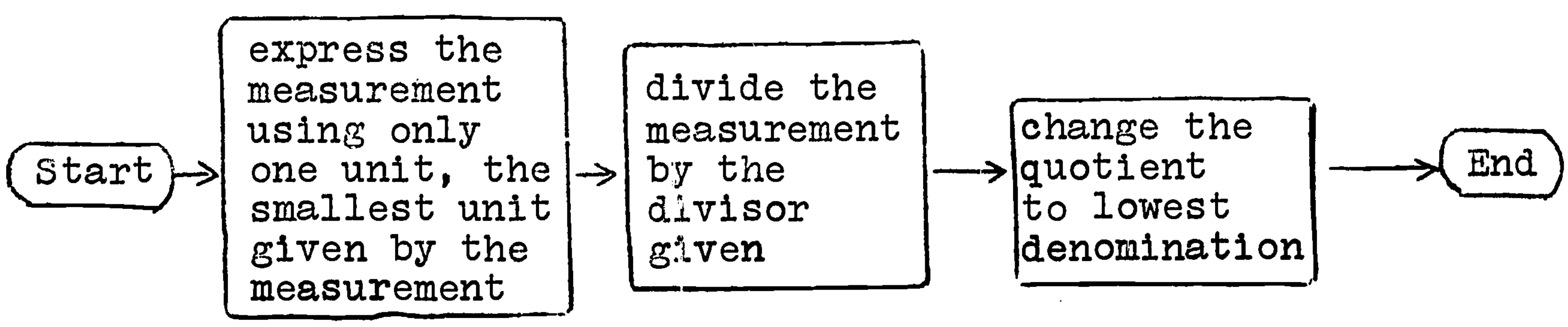


If you look at the flow-charts on pages 66 and 67 it appears that multiplying first and then changing to lowest denomination is by far the easier method. Perhaps it is a bit easier, but not very much easier. People who use measurements frequently, like carpenters and plumbers, use both methods. Some people claim that the method on page 66 is easier and others insist that the method on page 67 is easier. Since people disagree widely, you'll just have to decide for yourself which method you prefer.

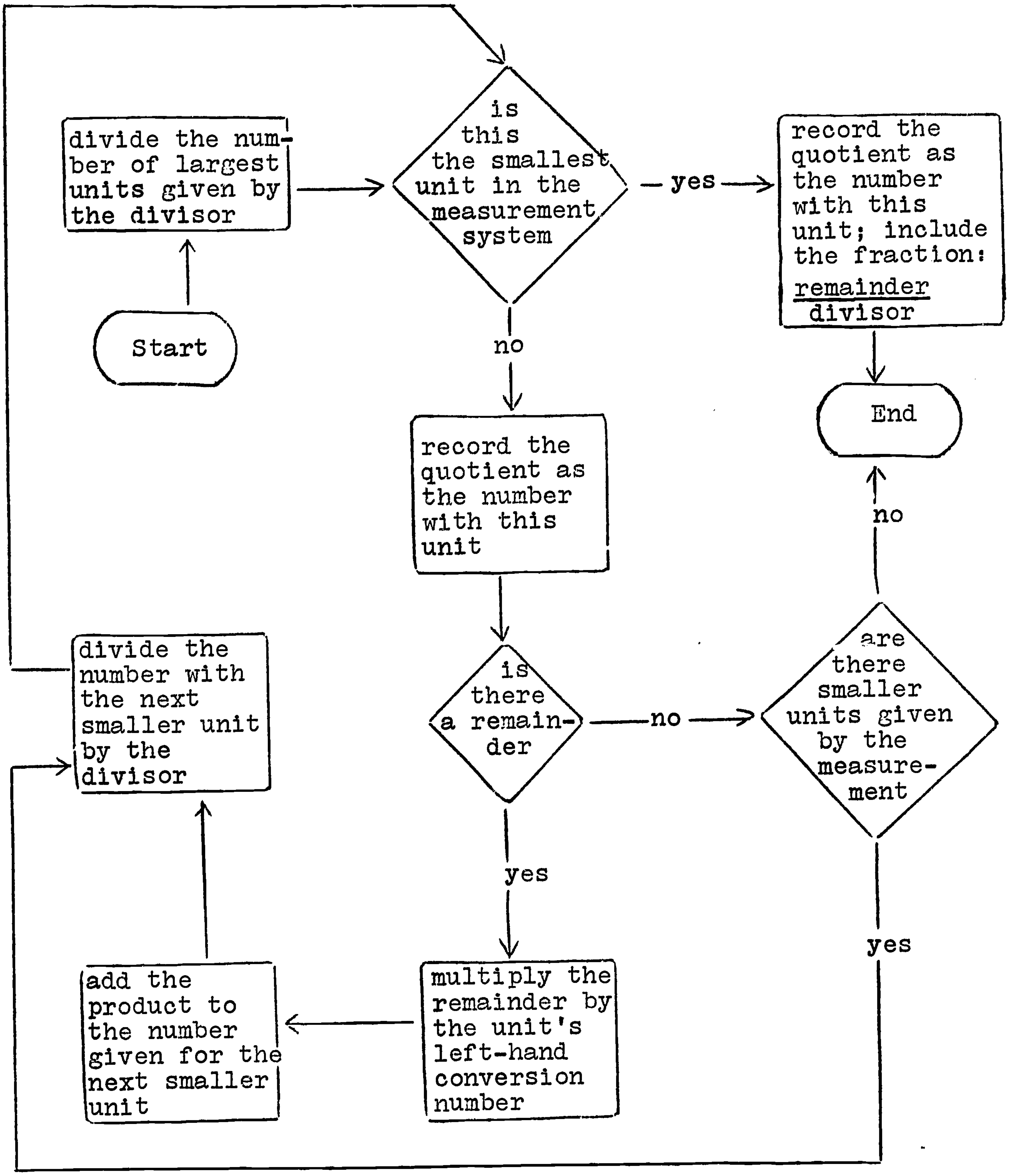
When you see the flow-charts for division, one will look much easier than the other, just as happened with flow-charts for multiplication. But with division most people choose the method with the longest flow-chart.

CLASS DISCUSSION After studying the next two flow-charts, see if you can think of reasons why the second method is usually prefered. Work some problems using both methods to see if this helps you get some ideas.

FLOW-CHART FOR DIVIDING MEASUREMENTS (Changing to lowest denomination after dividing)



FLOW-CHART FOR DIVIDING MEASUREMENTS (Changing to lowest denomination as you divide)



Perform the indicated operations:

(You will very likely want to use the information on page 47. If you think they would help, you may also use the flow-charts on pages 66, 67, 68, and 69.)

1. Multiply 2 feet, 5 inches by 6.

2. Divide 2 feet, 6 inches by 3.

3. Multiply 2 bushels, 3 pecks, 1 quart, 1 pint by 8.

4. Divide 6 square feet, 100 square yards by 25.

5. Divide 1 ton, 1 pound, 1 ounce by 2.

6. Divide 50 ares, 50 centiares by 100.

7. Multiply 1 pound, 7 ounces, 20 pennyweights, 4 grains by 5.

8. Find the average of the following measurements:

68 yards, 5 inches

5 feet, 1 inch

7 yards

2 yards, 2 feet, 2 inches

63 inches

9. CLASS DISCUSSION 2 yards, 68 feet, 40 inches is clearly not in lowest denomination. Divide this measurement by 5 without first changing it to lowest denomination. (You may use the flow-chart on page 69.)

(a) Is the answer correct?

(b) Is the answer in lowest denomination?

(c) Can you think of any reason why it is better to first change to lowest denomination before you divide? (Don't be alarmed if you can't come up with any very good reasons.)

10. CLASS DISCUSSION List as many situations as you can in which you might be asked either to multiply or divide measurements.

Chapter SEVEN
(Changing from one measurement systen to another)

There is one rather nasty problem that comes up because of all the different systems that people use around the world. Sometimes people who use mainly one system want to trade or communicate with people who usually use a different system. In such cases it is necessary to change the measurements. For example, a man in the United States might buy rope from an English businessman. He would very likely buy the rope by the meter. But to sell the rope in his store in the United States he would want to measure it in yards. If he has a box which he knows contains 50 meters of rope, what must he write on that box so that his customers in the United States will know how much rope there is in the box? One thing is certain: if he doesn't want to be bothered by a lot of questions about how much rope there is in the box, he should write the measurement using yards, feet, or inches.

Things here are not as nice as they were when we worked in a single measurement system. Inside most measurement systems there always happened to be some nice whole number of smaller units in the larger unit. For example, there are 12 inches in a foot; there are 3 scruples in a dram; and there are 1728 cubic inches in a cubic foot. Things would be much messier if it turned out that there were 13.672 inches in a foot; or $2\frac{1}{4}$ scruples in a dram. But if you take a meter stick and a yard stick you will see that there are 39.37 inches in a meter. Or, if you like, there

there are 3.28 feet in a meter. Looking at yards and meters, there are 1.09 yards in a meter. Somehow we just can't seem to find a unit we use in stores in the United States which will fit nicely into the meter.

There doesn't seem to be any way out; if we want to change meters to yards or yards to meters, we'll just have to use messy numbers like 1.09.

CLASS DISCUSSION

(a) In changing meters to yards, which number will be larger, the number of meters or the number of yards?

(b) What would we have if we wanted to change 1 meter to yards?

(c) What would we have if we wanted to change 2 meters to yards?

(d) Try to state a rule for changing meters to yards.

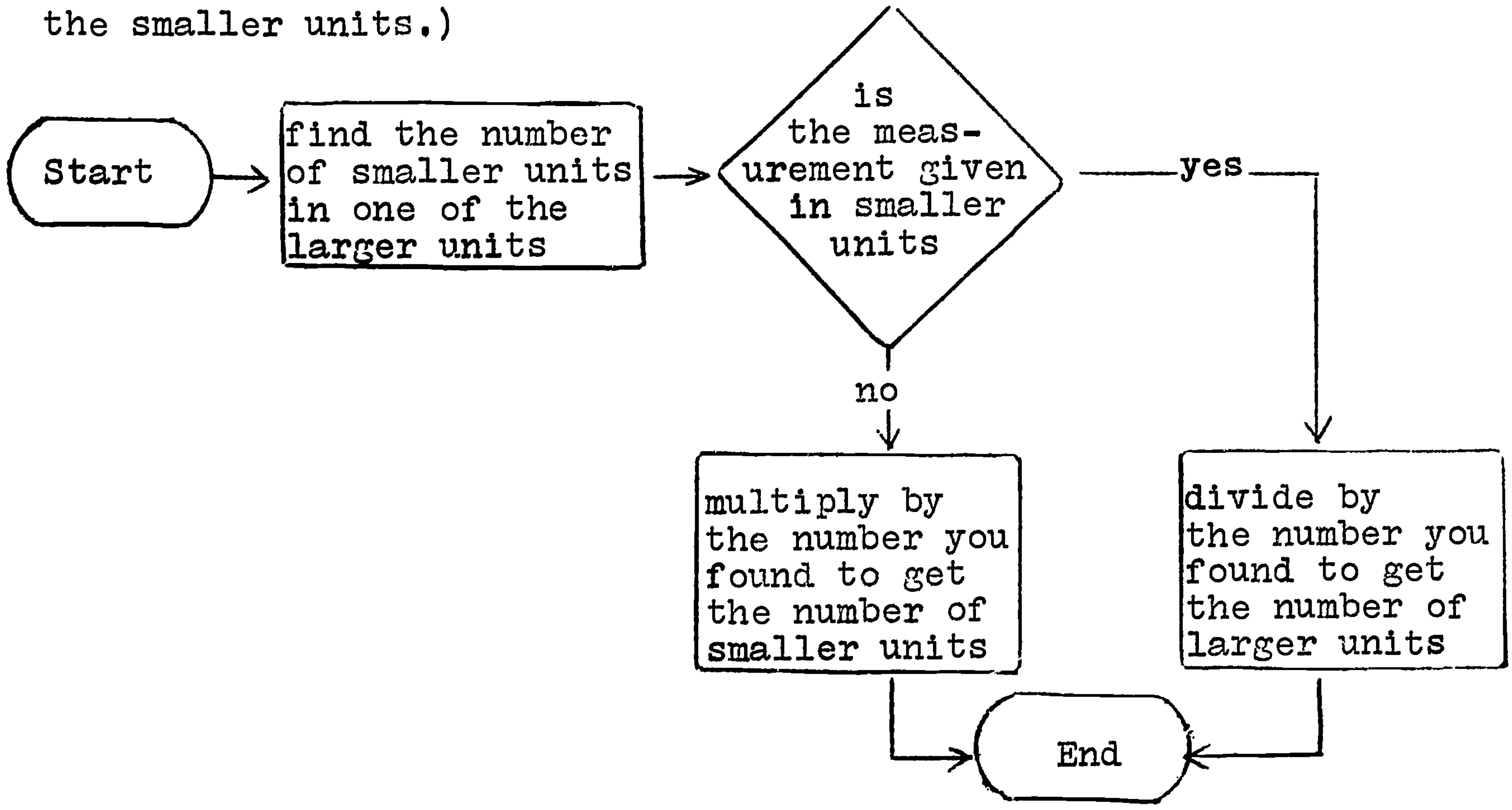
(e) What would we have if we wanted to change 1.09 yards to meters?

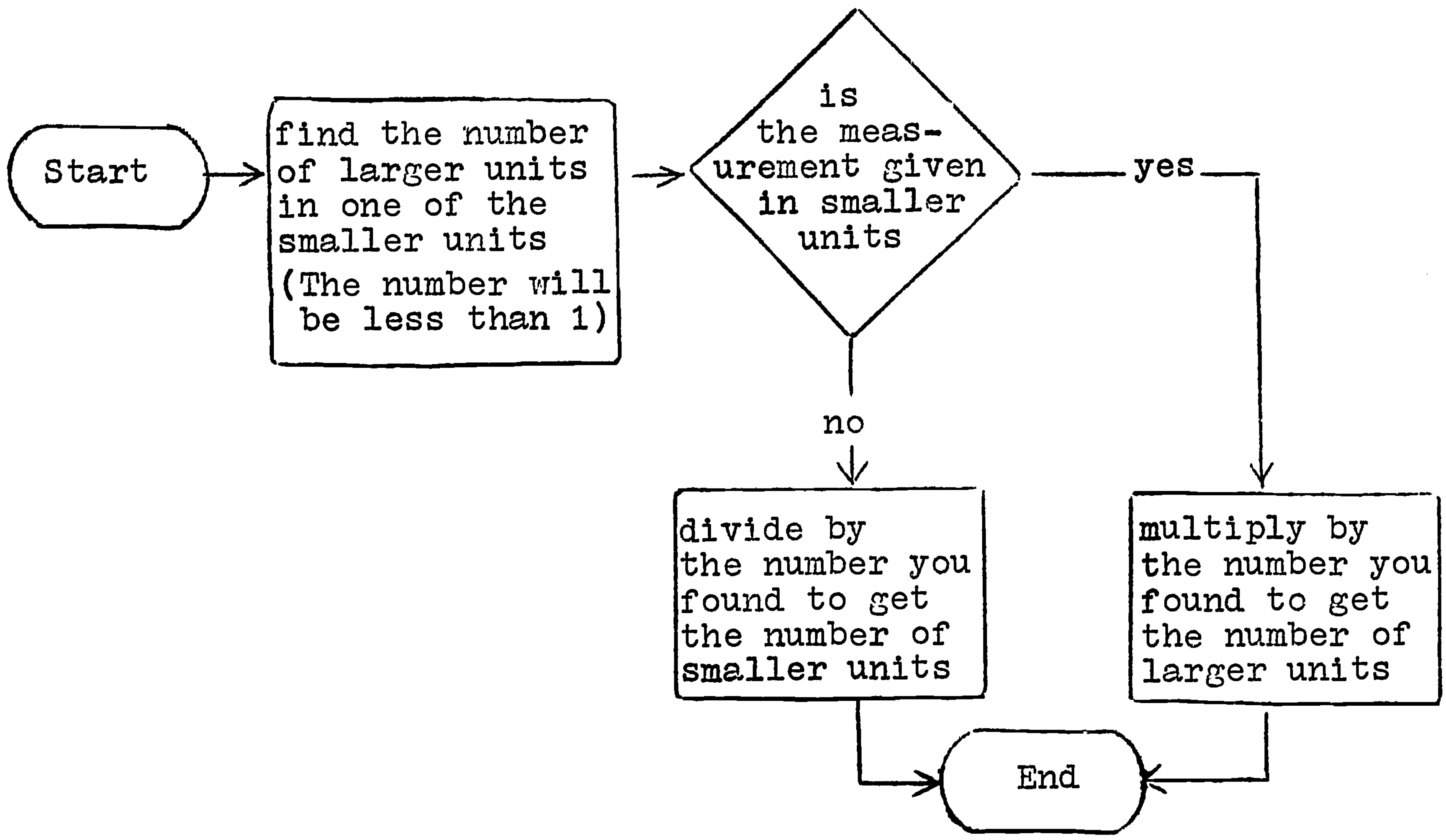
(f) What would we have if we wanted to change 3.27 yards to meters?

(g) Try to state a rule for changing yards to meters.

(h) One thing is sure to be true whenever we change from one type of unit to another: one of the units will be larger and the other will be smaller. (If this weren't the case, there would be absolutely nothing to do.) Another thing will always be there when we want to work with two units: there will be a number which tells us how many of the smaller units there are in the larger units. The only unpleasant thing about this chapter is that the number isn't likely to be a well-behaved whole number. It will usually be something messy like 3.28 or 1.09.

As a class, decide whether or not the following flow-charts make good sense. (The second flow-chart is for cases in which it is easier to find out how many larger units there are in one of the smaller units.)





It turns out that either of the flow-charts on the last page will work. Which one to use is always determined by which of two numbers is given. If you are given the number of smaller units in one of the larger units, you would use the flow-chart on the top of the page. If you are given the number of larger units in one of the smaller units, you would use the flow-chart on the bottom of the page.

If we wanted to suffer, we could try to think of ways for changing from one measurement system to another and changing to lowest denomination all at once. But all the people most of us will work with will do the following:

(1) Express the measurement using only one of the units from the system in which the measurement is given.

(2) Find either the number of larger units in one of the smaller units, or the number of smaller units in one of the larger units.

(3) Change the measurement to the other system. (By using one of the two flow-charts on page 73.)

(4) Change the measurement to lowest denomination.

It usually isn't necessary to first sit down and measure to find out how many smaller units there are in one of the larger units, or how many of the larger units there are in one of the smaller units. These facts can be found in the dictionary, an encyclopedia, or in a table in mathematics reference books.

Change each of the following measurements from the system given to the system asked for. Change the new measurements to lowest denomination only where you are asked to do so.

1. Change 39 feet to meters. (3.28 feet = 1 meter)

2. Change 6 links to meters. (1 link = .305 meters)

3. Change 5 pints to cubic inches. (1 pint = 33.6 cubic inches)

4. Change 200 cubic inches to pints.

5. Change 68 square inches to square centimeters.
(1 square inch = 6.45 square cm.)

6. Change 2 yards, 2 feet, 2 inches to the metric system.
(You may use the information on page 47, and you may like to know that 1 inch = 2.54 centimeters.)

7. To prove that measurement is really all fouled up, it turns out that if we are talking about what is called "dry measure," then 1 pint = 33.6 cubic inches. On the other hand, if we are talking about what we call "liquid measure," then 1 pint = 28.9 cubic inches. Supposing we had a strange substance which was neither dry or liquid. We wouldn't know if we should use "dry measure" or "liquid measure," If we knew that we had 5 pints using liquid measure, how many pints would we have if we wanted to use dry measure? (Don't feel sad if this problem seems terribly difficult. It is a truly nasty problem, but it is a type of problem which sometimes has to be solved.)